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Integrated Skills in the Secondary Mathematics Curriculum

Ana Teller

University of Mary Washington

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Integrated Skills in the Secondary Mathematics Curriculum

Ana Teller

University of Mary Washington

Applied Research

Dr. Beverly Epps

Summer 2013
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Introduction

Incorporating Integrated Learning into the Mathematics Classroom

No matter what a subject demands of students, critical thinking, problem-solving, collaboration, or understanding of the world around them will be necessary skills to have in order to succeed in a rapidly changing landscape of technology, global connectedness and demanding work places. Since current state standards do not meet these needs, the curriculum and teacher training must be adjusted. Collaboration must take place between content area teachers to connect information from one subject to another, and show students how to apply these skills outside of a specific domain.

Problem Statement

Skills such as critical thinking, problem solving, collaboration, overcoming failures, and global awareness are vital for students to learn before entering college or the work force. With careful planning, these skills can be embedded in the K-12 curriculum. Unfortunately, the current trend in education in the United States is focused on content knowledge rather than these so-called “soft skills.” As a result, we are graduating students who are unprepared for the changing demands of a global society in which future jobs have not yet been imagined.

Rationale

In most classrooms, students are being prepared to pass an annual standardized test by showing the knowledge they have learned throughout the year, beginning, for most states, in the third grade. This knowledge is often learned through the core subjects in isolation, dispersed from the teacher with the expectation that the students will memorize all of the pertinent facts or solve predetermined types of problem. Yet, when students leave high school and college to enter the workforce, they will be asked to apply this disconnected knowledge to their job, and life;
something they will not have had the chance to practice. Real life is not made up of multiple choice questions, practiced repeatedly throughout a year’s time; life requires critical thinking, creativity, ingenuity, collaboration, willpower, courage, and global-awareness. With a little effort and understanding, integrated skills can be taught in the classroom, making the required material more enjoyable and applicable for the students.

**Research Questions**

1. What skills are important for students to learn in their secondary education?
2. Why is it important to incorporate integrated skills in the curriculum?
3. How do schools ensure integrated skills are being taught in the classroom?

**Literature Review**

**Integrated Skills Defined**

Often called 21st-century skills, integrated skills define a set of abilities in increasingly high demand by the workforce, and the current world in which we live. Skills such as critical thinking, creativity, problem solving, collaboration, communication, global awareness, resilience to failure, and persistence are skills which are difficult to measure in a classroom setting, but are of the utmost importance to the future of our children, our society, and our world. A number of authors agree that our current educational system is one based on the 19th century, and that it is time our students, classrooms, and teachers catch up to the current demands of the world (Luterbach & Brown, 2011; Stevens, 2011; Wagner, 2012; What is 21st Century Education, 2008).

Marc Prensky calls the current generation of students “digital natives” (as cited in Wagner, 2012, p. xvi). Students now are accustomed to having electronic devices within reach almost every moment of their lives; except when they walk into most schools. These digital
natives, who are learning in a different way and have access to ideas and information within seconds, are being taught and tested as though none of this technology exists. However, use of technology is only a small piece of what makes up integrated skills. Critical thinking and problem solving are often thought of as skills learned in the mathematics classroom. However, as Taylor (2011) points out, these skills, along with creativity, are necessary for students in her art classroom, as well as all other classes, and can be taught in numerous ways (p. 22).

Collaboration, communication, and global awareness are becoming more and more important, especially the latter, as people in countries around the world are able to connect in seconds through technology. Resilience to failure and persistence are not often labeled as “skills,” yet they are imperative to accomplishing one’s goals. As Wagner (2012) has found in his research, “…qualities of innovators that I now understand as essential [are] …perseverance, a willingness to experiment, take calculated risks, and tolerate failure” (p. 12). Gasser (2011) explains that one of the realities of problem solving is failure, “or something uncomfortably close to it” (p. 112). Being able to be comfortable with failure, and use it as a learning experience is a skill that can be applied to many parts of a student’s life.

Whether these skills are called 21st-century skills, or integrated skills, and whether they are being taught in an art class, a science class, or a mathematics class should be of no consequence. What is important is that our students are learning them, being challenged in the process, and using their success as a motivation to better themselves as individuals, and members of our society.

Why Integrated Skills Matter

Societal demands are ever-changing, and students need integrated skills, many which are not currently being taught, to be a competitive force in the future. Wagner (2012) cites the
INTEGRATED SKILLS IN THE SECONDARY MATHEMATICS CURRICULUM

economy, social challenges, the rise in unemployment rates of young people, and an increase of outsourcing jobs to other countries as the main reasons why students should be learning 21st century skills (pp. 1-6).

Educators need to focus on incorporating integrated skills into every facet of education. Gasser (2011) reminds readers that graduating seniors will be entering jobs that did not exist when they first entered college (p.108), and Wagner (2012) cautions that, “Increasingly in the twenty-first century, what you know is far less important than what you can do with what you know. The interest in and ability to create new knowledge to solve new problems is the single most important skill that all students must master today” (p.142).

With the introduction of the No Child Left Behind Act, and, currently, the Obama administration’s education initiative, Race to the Top, high-stakes standardized testing has become the culminating experience for most students at the end of each school year, beginning in the third grade. Briggs (2013) cautions readers about focusing on content knowledge alone, “…today’s job landscape is changing so fast, and because high-paying, middle-skilled vocations are fewer and farther between, it is absolutely imperative for young professionals to be able to solve problems creatively and think critically” (Introduction section, para. 25). McCollum (2011) also warns of the perils of high-stakes standardized tests, saying they should be avoided, and instead, an assessment should be driven by the curriculum and integrate students’ experiences. The author goes on to state, “A students knowledge of a specific subject is important, but not the only piece of the puzzle necessary for success” (p. 5).

Unfortunately, standardized testing is what drives the current curriculum, rather than the other way around, and teachers’ hands are often tied as to what they can cover and teach in a single year. Wagner (2012) describes a teacher, Amanda Alonzo, who skillfully guides her
students in problem solving tasks, rich science experiments, and instills in them a love for learning; however, this all takes place on her own time, outside of her regular class periods. Alonzo talks about the required content knowledge for her state standardized tests; “They have to know that mitochondria make energy. Whereas in seminar [outside of class], I am teaching them how to figure out that mitochondria make energy. [This is] problem solving” (as cited in Wagner, 2012, p. 148). According to Luterbach and Brown (2011), some schools are infusing 21st-century skills into the curriculum, yet continue to assess them with 19th century practices – i.e., standardized tests (p. 23). However, when a teacher’s job potentially depends on his or her students’ success on the state tests, veering from the given curriculum is a daunting decision. Nevertheless, given the skills needed in today’s society, it must somehow be made a priority.

To be competitive in the ever-changing workforce of our current and future world, students must master these integrated skills by the time they enter college, trade school, or the workforce. These are the skills that matter most to leaders in for-profits, nonprofits, and military, says Wagner (2012, p. 12). According to Briggs (2013), “To get hired at Google, Microsoft, BBC News…– or to invent a new job in ten years – students need to spend more time using their skills than measuring them” (Students As Guinea Pigs section, para. 3). With the current focus on content rather than skill, Taylor (2011) asks the question, “…are we preparing students to function as human beings, or just as flesh-and-blood versions of a hard drive?” (p. 26).

**Schools Must Ensure Integrated Skills Are Being Taught**

Century after century, curriculum is revisited, studied, and altered in an attempt to meet the needs of current society. According to Rotherham and Willingham (2009) three primary components are required in order to successfully incorporate 21st-century learning into the
classroom: a complete program where “content is not shortchanged for an ephemeral pursuit of skills,” teacher training and, new assessments which “can accurately measure richer learning and more complex tasks” (p. 17).

**Curriculum.** A 21st-century curriculum must prepare students for jobs that have not yet been imagined. This may sound like an impossible task. However, as Rotherham and Willingham (2009) point out, this is nothing new; many of these “21st-century skills” have been required, and learned, for centuries. Abilities such as critical thinking, problem solving, global awareness, and information literacy have been crucial throughout history to aide in the development of the human race (p. 16). Yet the educational pendulum has currently swung to favor information over skills. Today students across the country are being prepared, all year long, to pass a state standardized test. Such tests will not assess integrated skills, yet it is the culminating activity for students in grades three through 12 in core subjects. As Taylor (2011) points out, “[i]t is a fact that we too often teach students to perform without them actually learning anything” (p. 22). Gasser (2011) compares the United States, where “the math curriculum…is a mile wide and a foot deep,” to other countries where less content, delivered more effectively, has proven to be beneficial in giving students the tools to apply their knowledge rather than recite it (p. 111). Taking the time to allow students to apply abilities such as problem solving and collaboration may eliminate the need to cover such a vast number of topics in a shallow way, and will impart these integrated skills in a natural way.

Curriculums in pursuit of integrated learning must include rich tasks that require problem solving, student-centered methods, problem-based learning, and authentic real-world problems (Rotherham & Willingham, 2009, p.19). According to Lockhart (2002), “The main problem with school mathematics is that there are no problems” (p. 9). Most of what passes as “problems” are
simply exercises in using a given algorithm. A real problem, according to Lockhart, “is something that you don’t know how to solve” (p. 9). However, when faced with a real problem, students must employ skills, such as critical thinking, collaboration, initiation, creativity, and imagination.

In addition to learning how to think critically and solve problems, integrated skills require that students be able to handle challenges and failures. Gasser (2011) cites that in other countries, such as Taiwan and China, students are often asked to solve math problems in front of the class before knowing yet if their solution is correct. Failure in these countries is not seen as a negative; it is seen as a chance for students to learn from their mistakes, and persevere to get it right the next time (p. 112). So often in the United States we encourage competition in which failure means you have lost, and the “game” is simply over. This is not a true reflection of life outside of sports or school. Children need to be unafraid of challenges, and the failures that come along with them.

By simply changing how students are asked to solve problems, integrated skills are naturally incorporated into the curriculum. This, however, does not mean that the current instruction is thrown out. There is still a need for content knowledge and process. Lockhart instructs, “Give your students a good problem, let them struggle and get frustrated. See what they come up with. Wait until they are dying for an idea, then give them some technique. But not too much” (p. 9).

**Teacher training.** As with any change in curriculum or new demands of teachers, there is a need for training and adjustment. Rotherham and Willingham (2009) cite that the majority of teachers favor student-centered methods, such as problem-based learning, to work on authentic problems and engage with the community. However, teachers are not using such
methods (p. 18). The authors go on to say, “…we don’t yet know how to teach self-direction, collaboration, creativity, and innovation the way we know how to teach long division” (2009, p. 17). Current teachers are often handed a pacing guide at the beginning of the year, and it is used as a checklist of topics to be covered, ones which will inevitably be seen on the standardized tests at the end of the year. In order to incorporate integrated learning into the curriculum, teachers need to be given guidance if it is something they are not comfortable incorporating on their own.

Implementing authentic tasks which require critical thinking, feedback, and guidance is demanding work for the teacher. It requires teachers to let go of the reins, so to speak, and be flexible and knowledgeable in their content area in order to allow the learning to happen in a fluid environment (Rotherham & Willingham, 2009, p. 18). In a math classroom, simply giving word problems is not sufficient. Luterbach and Brown (2011) argue that teachers need to also possess many of the same integrated skills that will be required of their students: content knowledge, communication, problem-solving, critical thinking, collaboration, and creativity. In addition, continual reflection is necessary in order to successfully incorporate these skills into their lessons (p. 20).

It should not be assumed that all teachers are prepared to deal with the extra time spent planning lessons, assessing, and implementing potentially “high-energy” instruction in order to incorporate 21st-century learning into their curriculum. However, if teachers are trained in classroom management, lesson development, and assessment to specifically implement more student-centered activities that focus on problem-based learning, the rewards could be endless. Lockhart (2002) talks about allowing his geometry students to work through a proof in their own words, and in their own way, which takes a considerable amount of energy for both the student
and the teacher, yet when shown the end result he remarks, “I’m not sure who was more proud, the student or myself” (p. 22).

Assessments. Not only do student assessments need to change in order to effectively gauge how well students are learning integrated skills, but assessments of teachers and curriculum need to be adjusted as well. Briggs (2013) found that in other nations such as the Netherlands, when changing their curriculum, outside inspectors do extensive follow-ups looking at more than simply test scores to evaluate any recent changes. However, in the United States, it is not until many classes of children have gone through our system after a major change, such as No Child Left Behind, that we find, in hindsight, a particular policy was not successful (Students As Guinea Pigs section, para. 2). Koretz says that this is “…a political problem because we lack information that we could use to better serve children. [Yet] it’s an ethical problem because children are not consenting adults” (as cited in Briggs, Students As Guinea Pigs section, para. 2).

According to Rotherham and Willingham (2009), the potential exists today to assess not only a student’s content knowledge, but thinking skills. The authors go on to idealize the tests, saying, “…a truly rich assessment system would go beyond multiple-choice testing and include measures that encourage greater creativity, show how students arrived at the answers, and even allow for collaboration.” However, creating an assessment of that scope on a large scale would inevitably be challenged by the cost (p. 18).

However, as Harris, P., Smith, and Harris, J. (2011) remind readers, assessments are what often drive what is taught in the classroom, whether intentional or not, and by “ignoring attributes that they can’t properly assess, standardized tests inadvertently create incentives for students to become superficial thinkers – to seek the quick, easy, and obvious answer” (p. 36).
While the cost of such assessments may be high, if school leaders and officials deem integrated skills as important skills for our students to learn, the assessments must change in order for the curriculum to truly transform.

**Conclusion**

In a society vastly different from the one most teachers went to school in, it is a challenge to envision what skills, exactly, our students will need to succeed. The current generation may not have the ability to imagine jobs that have not yet been created, but it is imperative that today’s school system find a way to give the next generation more skills to adapt to ever changing circumstances in a rapidly changing world. Current curriculums are driven by standardized tests, and until these standards are modified or changed, the curriculum will remain focused on content knowledge rather than ability. It is vital that our students have classes which balance both content knowledge and integrated skills. Teachers must make an effort to collaborate with each other within their curriculum constraints. They must be an example to their students of life outside the classroom by modeling the skills required for everyday life. While it is possible to incorporate integrated learning into the current curriculum, it requires time, energy, and patience. However, the alternative – graduating students unprepared to one day lead our society – should be unimaginable to teachers, administrators, schools, and policy makers.

**Application**

According to Jensen, “By immersing [students] brains in problems that contain much information to sort through and many components…students will be able to create more connections to past understanding and current applications. They will also attach more meaning to any learning that takes place throughout the problem-solving process” (as cited in Gasser,
Regardless of the time it takes for policies to change, curriculums to be revisited, and assessments to be made more authentic to learning, there are changes that can be made immediately in the mathematics classroom to further develop students’ integrated skills, and still ensure they are learning the material required for the state standardized test.

However, simply infusing the mathematics curriculum with rich tasks, problem-based learning, collaborative projects, and real-world applications will do nothing for the students if they do not also enjoy participating in such activities. Jensen notes, when learning is a joyful experience, the brain retains more information (as cited in Gasser, 2011, p. 113). According to Lockhart (2002), it seems as though teachers are trying too hard to make the required topics relevant, “You think something practical like compound interest is going to get [the students] excited? People enjoy fantasy, and that is just what mathematics can provide – a relief from daily life, an anodyne to the practical workaday world” (p. 9). Lockhart goes on to argue that simply “doing math” is the most elegant way to incorporate 21st-century skills. When students are given the autonomy to make math their own, they get excited about it, and delve into the problem-solving and creative process, which is true mathematics. Lockhart warns, “If you deny students the opportunity to engage in this activity – to pose their own problems, make their own conjectures and discoveries, to be wrong, to be creatively frustrated, to have an inspiration, and to cobble together their own explanations and proofs – you deny them mathematics itself” (p. 5).

In the following appendix, readers will find rich tasks requiring the use of integrated skills that incorporate the Virginia Standards of Learning for 8th grade mathematics. These lessons are meant to be used in place of textbook “drill and kill” problems, at the discretion and comfort of the teacher, and may be used to combine multiple topics.
References


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**Archimedes’ Box**

Length of activity (based on 50-minute periods): 1-3

**Objectives:**
- Students will employ problem solving to create Archimedes’ Box
- Students will apply formulas for area, perimeter, and the Pythagorean Theorem to solve problems.

**Leading Question(s):**
- How many ways can you solve Archimedes’ Box?
- What other information can we gather from this ancient puzzle?

**Integrated Skills Covered:**
- Problem Solving
- Collaboration (if in pairs/groups)
- Persistence

**Mathematics SOL Strand Covered:**
- 8.10 Geometry
- 8.11 Geometry
- 8.16 Patterns, Functions, and Algebra

**Lesson (see also attachments/handouts):**

**Students’ Responsibility**
- Spend some time looking at Archimedes’ Box.
- Brainstorm the various ways you will solve this puzzle – you may want to talk with others in the class.
- Be sure to record your work, and any interesting findings.

**Teacher’s Responsibility**

**Option 1**
- Give students the cut-out pieces of Archimedes’ box, and have them put them back together into a square in as many ways as possible.
- This could be done individually or in pairs

**Option 2**
- Have students classify the various polygons that make up the Box.

**Option 3**
- Using the Pythagorean Theorem, students find the perimeter of each triangle in the Box.
- Compare answers and discoveries with the class.
- See attached handout.

**Option 4**
- Students find the area of each box.
- Compare answers and discoveries with the class.

**Option 5**
- Have students give the equation of each line after creating the Box.

**Assessment Options:**
- Have students neatly record their work for the perimeter and areas of Archimedes’ Box.
- Have students present their method for solving to the class.
- Informally assess students as they create the Box from the various pieces.

Lesson adapted from Dr. Don S. Balka, Saint Mary’s College
Archimedes’ Box:
Check Out These Spinners

Probability

Length of activity (based on 50-minute periods): 2 days

Objectives:
- Students will discover the difference between experimental and theoretical probability.
- Students will discover the multiplication principle for independent events.

Guiding Question(s):
- Which is more reliable, luck or probability?
- What should happen to our experimental data as we gather more and more data?

Materials Needed:
- Paperclips
- Spinner templates
- Questions handout
- Extra paper to record spins

Integrated Skills Covered:
- Critical Thinking
- Communication
- Collaboration
- Persistence

Mathematics SOL Strand Covered:
- 8.1 Number and Number Sense
- 8.12 Probability and Statistics
- 8.13 Probability and Statistics

Lesson (see also attachments/handouts):

Students’ Responsibility:
- Read directions carefully.
- Answer questions #1 and #2 BEFORE you begin spinning.
- Understand how the game is played before you begin.
- Once you have “played” 50 games, give your teacher your results in a fraction, decimal, and percent.
- Record other pairs’ data, and continue answering questions once all class data is available.

Teacher’s Responsibility:
- Show students how to create the spinners out of paper clips.
- Talk with the class about the probability of each spinner (should be review).
- Go over the rules of the “game” with the class (both spinners must land in the shaded area to “win”).
- Have students pair up, and work through the questions.
- Once pairs have their 50 spins and “wins” recorded, write them on the board.
- Students will need to be reminded to read the questions carefully to discover the multiplication principle.

Assessment Options:
- Collect questions and work done by pairs
- Informal assessment while students work
- Exit ticket or quiz with multiplication principle and independent/dependent events

Lesson from NCTM, *Navigating Through Probability in Grades 9-12*
Check Out *These Spinners*

Consider the pictured spinners:

Suppose that you win a game if you spin each of these spinners one time and both arrows land in shaded areas.

1. What do you think your chance of winning this game would be? ______ Why?

2. If you played this game 50 times, about how many times would you expect to win? ______ Why?

3. Working with a partner, use spinners like those pictured and gather data on 50 games. What percentage of the time did both spinners land in shaded areas? ______

4. Gather data from enough other pairs of students to give you results from hundreds of games with the spinners. What percentage of the time did both spinners land in shaded areas? ______ How does this percentage compare with your prediction in number 1?

5. a. If you played the game 2000 times, how many times would you expect the first (one-half shaded) spinner to land in the shaded area? ______ Why?

   b. About how many of those spins in the shaded area would you expect to be paired with another spin in the shaded area on the second (one-quarter shaded) spinner? ______ Why?
Check Out These Spinners (continued)

Name ________________________________

6. The number that you wrote in 5(b) of number 5 is the total number of games out of 2000 in which you would expect both spinners to land in shaded areas. What percentage of the 2000 games is that number? ______ What is the probability that both spinners will land in shaded areas? ______

7. What is the probability that the first spinner will land in the shaded area? ______ What is the probability that the second spinner will land in the shaded area? ______ How are these individual probabilities related to the probability that you gave in number 6 that both spinners will land in shaded areas?
Google Earth Tours

Length of activity (based on 50-minute periods): Multiple days, will vary

Objectives:

- Students will use a guided Google Earth tour to learn about different parts of the world through mathematics.
- Topics such as exchange rates, percent of change, area and perimeter measurement.

Leading Question(s):

- What is a fractal coast?
- How much would a banana cost in Zimbabwe?
- How large is a crop circle?

Materials Needed:

- Computers with internet access; 1:1 or 2:1 student to computer ratio
- KMZ files downloaded before lesson from http://www.realworldmath.org/lesson-downloads.html
- Handouts where necessary

Integrated Skills Covered:

- Global Awareness
- Problem Solving
- Communication
- Collaboration

Mathematics SOL Strand Covered:

- 8.3 Computation and Estimation
- 8.11 Geometry
- 8.13 Probability and Statistics
- 8.14 Patterns, Functions, and Algebra

Lesson (see also attachments/handouts):

Students’ Responsibility

- Read directions carefully, and completely.
- When moving around the Google Earth tour, remember to double click on the next step (single click will not zoom the map).
- Be sure to show your work, nothing will be saved in your tour.
- Talk with your partner or a neighbor if there are questions.

Teacher’s Responsibility

- Before giving students computers, review guidelines for proper use.
- Show on a projector what one of the tours looks like, and how to navigate through Google Earth.
- Students individually or in pairs may be assigned the same tour, or different tours depending on the topics needing to be covered.
- Once in pairs, students will self-guide through the tour, answering questions as they come up.
- Remind students to show work and record answers as nothing will be saved in Google Earth.

Assessment Options:

- Will vary depending on tour used.

Lesson adapted from http://www.realworldmath.org/
Example of a question on the Crop Circles measurement tour:

The outer wheels of a central pivot irrigation system travel the greatest distance.

If this unit in Australia makes 3 complete rotations in one day, how many kilometers would its outer wheels travel?

Use the Historical Imagery tool and set the slider to the date of 8/31/2006.
How Many Texts Do You Send In A Week?
Data collection and interpretation

Length of activity (based on 50-minute periods): 1 day (with prior weekend homework)

Objectives:
- Students will collect individual data, plot data, and interpret a line of best fit to predict their texting habits over the period of a week, month, or even year.

Leading Question(s):
- How many texts do you think you send in a week? A month? A year?
- How is this data reliable? Unreliable?

Integrated Skills Covered:
- Problem solving
- Collaboration
- Communication

Materials Needed:
- Instructions
- Text Text Text handout
- Graph paper
- Rulers

Mathematics SOL Strand Covered:
- 8.13 Probability and Statistics
- 8.17 Patterns, Functions, and Algebra

Lesson (see also attachments/handouts):

<table>
<thead>
<tr>
<th>Students’ Responsibility</th>
<th>Teacher’s Responsibility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Over a weekend, collect your texting data for one day (or more if desired).</td>
<td>Assign homework over a weekend for students to track their texting habits over a day’s time.</td>
</tr>
<tr>
<td>Be sure to collect your data carefully.</td>
<td>Come together as a class and talk about the differences in data (some students may choose to do a different topic than texting).</td>
</tr>
<tr>
<td>When you return to school you will plot and predict your texting habits for a week, month, and a year.</td>
<td>Pass out graph paper.</td>
</tr>
<tr>
<td>Aren’t you glad you don’t pay your phone bill?</td>
<td>Have students plot their data in such a way that they will be able to have room to extend their line of best fit (it is best to let them work this out on their own, rather than tell them exactly how to make their graph).</td>
</tr>
</tbody>
</table>

Assessment Options:
- Collect graphs and data to ensure correctness of work
Text Text Text Text Text Text Text Text Text Text Text Text Text Text Text Text

Name __________________________

**How much do you text?**

Over this weekend, you will collect data about your texting habits. Or, if you don’t text, something that you do a lot during the day (internet, video games, basketball, swimming...).

Record the start time for your data, then after each hour, record how many texts you have sent (not received). Try not to change your normal texting numbers – we want to know what really happens in a day – and maybe over the course of a year!

**Start time/date:**

<table>
<thead>
<tr>
<th>Hour</th>
<th>Number of texts sent in the past hour</th>
<th>Cumulative number of texts sent (like a running total)</th>
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<tbody>
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<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13??</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

If you would like to continue to collect data over a second day, record it on the back, and repeat the previous steps.
Measuring the Earth
The Noon Day Project

Length of activity (based on 50-minute periods): 3 class periods

Objectives:
- Students will apply their knowledge of similar triangles, angle measurement, and circumference of circles.
- Students will gain knowledge about a city in another country that they have paired with for the project.

Leading Question(s):
- Would you believe it is possible to measure the circumference of the earth, with only a yard stick to make a shadow, and a protractor to measure angles?

Materials Needed:
- "The Librarian Who Measured the Earth" by Kathryn Lasky
- Yard sticks
- Large paper
- Protractors
- Access to http://ciese.org/curriculum/noonday/
- Attached directions for groups
- Watches/clocks

Integrated Skills Covered:
- Problem Solving
- Critical Thinking
- Creativity
- Collaboration
- Communication
- Global Awareness
- Persistence
- Resilience to Failure

SOL Strand Covered:
- 8.3 Computation and Estimation
- 8.4 Computation and Estimation
- 8.6 Measurement
- 8.11 Geometry

Lesson (see also attachments/handouts):

Students’ Responsibility
- Listen carefully to the story of Eratosthenes
- Be sure you understand the directions – read carefully again with your group before going outside.
- Assign within your group a timer, a recorder, and a marker (to mark the end of the shadow). You may want to double up on the recorder if you have more than 3 people in your group.
- Once you get outside and set up, don’t forget to mark your base!
- Take shadow measurements according to the directions.
- Resume in classroom with group to discover the circumference of the earth!

Teacher’s Responsibility

Day 1
- To introduce the project, read “The Librarian Who Measured the Earth” by Kathryn Lasky to the class.
- Go through the directions with the class before breaking into pairs. Talk about the formulas which will be necessary to complete this assignment.
- As a class, choose your partner city from the list of participants on the website.
- Assign groups of 3-4 students.

Day 2 (Outside)
- Have students collect the necessary materials.
- Go outside to get set up approximately 15 minutes before the anticipated local noon.
- DON’T FORGET TO MARK YOUR BASE!

Lesson adapted from The Noon Day Project and the CIESE, http://ciese.org/curriculum/noonday/
**INTEGRATED SKILLS IN THE SECONDARY MATHEMATICS CURRICULUM**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>• Have students take shadow measurements every 2 minutes for</td>
<td>• Have students take shadow measurements every 2 minutes for</td>
</tr>
<tr>
<td>approximately 10 minutes before local noon, and continue</td>
<td>approximately 10 minutes before local noon, and continue</td>
</tr>
<tr>
<td>until shadow begins to grow longer again.</td>
<td>until shadow begins to grow longer again.</td>
</tr>
<tr>
<td><strong>Day 3 (Back in classroom)</strong></td>
<td><strong>Day 3 (Back in classroom)</strong></td>
</tr>
<tr>
<td>• Have students complete the guided handout in their groups.</td>
<td>• Have students complete the guided handout in their groups.</td>
</tr>
</tbody>
</table>

**NOTE:** This lesson needs to be completed at a very specific time of day, and may not be possible for all class periods, or students, to complete.

**Assessment Options:**
- Have students journal their ideas about how they will measure the earth, before and after the project
- Have students turn in all necessary work from the project
Measuring the Earth
In conjunction with the Noon Day Project

Names in group

Info from http://ciese.org/curriculum/noonday/

- You should try to get your measurement done no later than by the end of week 3. Our target is to do the measurements as close to the Equinox as possible but anytime during this two week span will be fine.
- Do the measurement when the sun is highest in the sky (at your local noon time.) – For us this is approximately 1:17pm
- To do this experiment you will need some materials to measure shadows accurately. Your “gnomon” will be a meter stick that is perpendicular to the ground. For your measurements to be accurate, it is critical that the meter stick be vertical. (Note the devices used below. Wind can be a major factor).
- It is helpful to have a piece of paper to note where the end of the shadow is. Also a compass will come in handy to determine in which direction the shadow falls. Since the edge of the shadow is "fuzzy" and the shadow is moving from west to east (northern hemisphere), you want to be careful in deciding where to place your mark.
- You may find it interesting that the shadow points towards the north. But does it point to true or magnetic north? A compass will come in handy to determine this (anyone bring their cell phone? You won't get in trouble – this time!)
- Take measurements every 2 minutes beginning at least 10 minutes before local noon which is the time that the sun is highest in the sky. (This will most likely NOT be 12 noon as indicated on your time measuring device (sometimes called a watch). You should note that when the sun is highest in the sky the shadow length is the shortest.
- Make a scale drawing of your stick and shadow. Complete the triangle and measure the sun’s angle with a protractor.

Material List:

- Meter stick (in place of this we are using the metal science thing that Mrs. Teller does not know the name of)
- White paper – long enough to trace a shadow from metal posts
- Rulers (in class only)
- Pencil/marker
- Protractor
- Watch with second hand (at least one per group)
INTEGRATED SKILLS IN THE SECONDARY MATHEMATICS CURRICULUM

Directions:

1. When you get outside, position your metal post so it is perpendicular, and place the white paper on the ground so you can see the shadow on it
2. Be sure to mark on your white paper where your stand is (you will need it to take measurements)
3. You will begin marking shadow lengths ASAP (hopefully by 1:07pm!), be sure to note the time for each one (this may be done directly on your white paper, next to the mark for the shadow length.
4. Before “local noon” your shadow lengths should be getting shorter
5. You will know we have reached “local noon” when your shadow length starts to get longer again (the shortest measurement will be the one we use)
6. Please be sure to leave your white paper with name, times, and shadow marks in the classroom for tomorrow.

Follow up (Next Day):

1. You should have approximately 12 measurements of your shadow (about 8 before local noon, and about 4 after)

   List the length of the shadow from each measurement here:

<table>
<thead>
<tr>
<th>Time</th>
<th>Shadow length (in cm)</th>
</tr>
</thead>
</table>

2. Height of metal post (in cm): ______________________________

3. Length of “local noon” shadow: ______________________________
   (This will be your shortest shadow)

Lesson adapted from The Noon Day Project and the CIESE, http://ciese.org/curriculum/noonday/
4. Scale drawing of metal post and shadow (and resulting triangle). PLEASE MAKE THIS AS ACCURATE AS POSSIBLE!! (If there is not enough room here, please do it on a separate sheet of paper). See example in #5.

5. Using a compass, measure the angle of the sun (example below).

**EXAMPLE:**

Our two sites will be at Manasquan, New Jersey and San Juan, Puerto Rico.

The next step is to figure out the sun angles at Manasquan and Puerto Rico. A simple method you can use is to create a scale model of your triangles and then directly measure the angle. In this case we could create a triangle for Puerto Rico with sides that are 3.53 cm and 10.0 cm. We could also use 3.53 inches and 10.0 inches. As long as the proportions are the same the angle should be the same. A protractor can be used to measure the angle.

The central angle (of the Earth) equals the difference between these angles. Using these measurements, the central angle is 38.85 - 18.59 or 20.26 degrees.

Lesson adapted from The Noon Day Project and the CIESE, [http://ciese.org/curriculum/noonday/](http://ciese.org/curriculum/noonday/)
6. Using your scale triangle above, measure the angle of the sun in Warrenton, VA: _________________

7. Angle of the sun in __________________________ : ________________

8. Resulting central angle: _________________________________

9. Auburn Middle School, Warrenton, Virginia Latitude/Longitude: W 77.7, N 38.7

   (Other city) ________________ Latitude/Longitude: _______

   North/South difference (in degrees) between the two cities: _________________________

   Each degree of latitude is about 111km apart. Distance apart in km: ________________
   (Please show work)

10. The distance between the two cities, found in question #9, is the length of the arc along the circumference of the earth. Using this information, we can find the rest of the circle, also known as the circumference of the earth!

   Brainstorm in your groups about ways you could find the entire circumference, given the distance of just a “slice.” Think back to the story of Eratosthenes!
Nice to meet you, Sandy, my name is _______________

Hurricane Tracking

Length of activity (based on 50-minute periods): 2 class periods

Objectives:
- Students will review graphing on a coordinate plane, as well as apply use of making a line of best fit, writing the equation of the line of best fit, and predicting data using the line of best fit.

Leading Question(s):
- How are hurricanes named?
- What can their paths show us?
- How fast is the wind of a hurricane?

Materials Needed:
- Attached handouts
- Tracking map
- Colored pencils
- Students in groups of 2-3

Integrated Skills Covered:
- Communication
- Global Awareness
- Collaboration
- Problem Solving

Mathematics SOL Strand Covered:
- Review of coordinate plane (7th grade)
- 8.13 Probability and Statistics
- 8.16 patterns, Functions, and Algebra

Lesson (see also attachments/handouts):

<table>
<thead>
<tr>
<th>Students’ Responsibility</th>
<th>Teacher’s Responsibility</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Read directions carefully.</td>
<td>• Introduce hurricane tracking, talk about any recent activity in your area, or prompt students to share stories about any recent hurricanes they remember experiencing.</td>
</tr>
<tr>
<td>• In groups, map the two hurricanes side by side, and answer the questions about each.</td>
<td>• Place students in groups of 2-3.</td>
</tr>
<tr>
<td>• Remember, check with your group first before you ask the teacher a question.</td>
<td>• Supply necessary material.</td>
</tr>
<tr>
<td></td>
<td>• Students should share 1 map among the group to ensure group participation.</td>
</tr>
<tr>
<td></td>
<td>• As students become stuck or ask questions, ensure that they have checked with the group first.</td>
</tr>
<tr>
<td></td>
<td>• Some answers may vary; be sure students are justifying their answers.</td>
</tr>
<tr>
<td></td>
<td>• Leave time between activities to talk as a class about answers, and clear up any confusion.</td>
</tr>
</tbody>
</table>

Assessment Options:
- Informal while students are working; walk around to check answers and see work
- Have each group present their findings, or a particular question
- Re-pair groups and have the new groups compare answers; have each student write a summary of similarities and differences between their original group, and second group.
Nice to meet you, Sandy, my name is ________________

From the New York Times, October 29, 2012
Sandy — which was reclassified as a non-tropical storm because of its unusual dynamics — came ashore at 8 p.m. in Atlantic City [October 29, 2012], carrying sustained hurricane-force winds of 80 mph or more and dangerous flood tides as high as 13 feet, the National Hurricane Center said.

Sandy strengthened in the morning, with maximum sustained winds reaching 90 miles per hour, up from 75 mph previously. As predicted, the vast storm — some 900 miles wide — began moving west from the ocean toward land.

In Washington, winds gusted as high as 60 mph.

Hurricane Tracking
The following table tells the location of the centers of two hurricanes: Humberto and Sandy. The positions mark the location of the hurricanes at midnight (when one day ends and another begins). Position 0 and position 1 mark the start and end of day 1. Then position 1 and position 2 mark the start and end of day 2, and so forth. Can you tell what position numbers define day 6?

<table>
<thead>
<tr>
<th>Position</th>
<th>Hurricane Sandy</th>
<th>Hurricane Humberto</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>12° N, 79° W</td>
<td>15° N, 65° W</td>
</tr>
<tr>
<td>1</td>
<td>16° N, 77° W</td>
<td>16° N, 68° W</td>
</tr>
<tr>
<td>2</td>
<td>20° N, 76° W</td>
<td>17° N, 71° W</td>
</tr>
<tr>
<td>3</td>
<td>26° N, 77° W</td>
<td>18° N, 74° W</td>
</tr>
<tr>
<td>4</td>
<td>28° N, 77° W</td>
<td>20° N, 75° W</td>
</tr>
<tr>
<td>5</td>
<td>32° N, 74° W</td>
<td>21° N, 76° W</td>
</tr>
<tr>
<td>6</td>
<td>35° N, 71° W</td>
<td>22° N, 77° W</td>
</tr>
<tr>
<td>7</td>
<td>41° N, 77° W</td>
<td>24° N, 79° W</td>
</tr>
<tr>
<td>8</td>
<td>42° N, 80° W</td>
<td>26° N, 79° W</td>
</tr>
<tr>
<td>9</td>
<td></td>
<td>29° N, 77° W</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>32° N, 71° W</td>
</tr>
<tr>
<td>11</td>
<td></td>
<td>35° N, 63° W</td>
</tr>
</tbody>
</table>

1. Plot the position points on the hurricane tracking map for both hurricanes. You may want to use different colors for different hurricanes. Important: Be sure to indicate the direction each hurricane is traveling.

Adapted from NCTM, May 2008, Mathematics Teaching in the Middle School
2. What countries do these two hurricanes affect? What do you observe about the direction they travel and the shape of their paths?

3. Compare and contrast the path of Hurricane Humberto with the path of Hurricane Sandy. In other words, how are their paths similar and how are they different?

4. How many degrees latitude did Humberto change between position 2 and position 3? Explain.

5. On which day(s) did Sandy have the largest change in longitude? Explain.

6. Which day(s) did Sandy have no change in latitude? Explain.

7. Which day(s) did Humberto have no change in latitude? Explain.

8. Which hurricane traveled the farthest over the first two days: Humberto or Sandy? Explain.
9. Decide which day Sandy traveled farther: day 2 or day 3. How do you know that this length is longer than the other? Justify your answer.

10. (a) Which single day did Humberto travel the farthest? Justify your answer.

   (b) How many miles did Humberto travel on that day? Justify your answer. (Hint: Use the scale on the map.)

11. (a) Which hurricane traveled the farthest in one day? Which day was it? How far did it travel? Explain your reasoning.

   (b) What was the average speed of that hurricane in miles per hour on that day? Explain your solution.

12. Latitude and longitude on a flat map divide the surface of the Earth into a plane that is similar to the Cartesian coordinate plane with its four quadrants.

   (a) In what quadrant on the Cartesian coordinate plane would the region in which you plotted the hurricanes belong? Explain your reasoning.

   (b) How would the positions for the hurricanes given with respect to north and west (i.e., 15° N, 65° W) be written in point notation (x, y) to graph on a Cartesian plane? Explain your reasoning, and provide examples.
math for real

"when will I ever use this?"

Jan A. Yow

Hurricane-Force Math

HURRICANE FRAN

(August 23—September 8, 1996)

<table>
<thead>
<tr>
<th>Date/Time (UTC)</th>
<th>Pressure (mb)</th>
<th>Wind Speed (kt)</th>
</tr>
</thead>
<tbody>
<tr>
<td>23/1200</td>
<td>1012</td>
<td>25</td>
</tr>
<tr>
<td>24/1800</td>
<td>1009</td>
<td>30</td>
</tr>
<tr>
<td>27/1200</td>
<td>1005</td>
<td>35</td>
</tr>
<tr>
<td>28/0000</td>
<td>1002</td>
<td>45</td>
</tr>
<tr>
<td>30/0600</td>
<td>991</td>
<td>65</td>
</tr>
<tr>
<td>31/0000</td>
<td>982</td>
<td>60</td>
</tr>
<tr>
<td>01/1200</td>
<td>982</td>
<td>70</td>
</tr>
<tr>
<td>03/1800</td>
<td>968</td>
<td>85</td>
</tr>
<tr>
<td>04/1200</td>
<td>956</td>
<td>105</td>
</tr>
</tbody>
</table>

HURRICANE CARLOS

(July 10–16, 2009)

<table>
<thead>
<tr>
<th>Date/Time (UTC)</th>
<th>Pressure (mb)</th>
<th>Wind Speed (kt)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10/6000</td>
<td>1007</td>
<td>25</td>
</tr>
<tr>
<td>11/1800</td>
<td>1000</td>
<td>45</td>
</tr>
<tr>
<td>12/0000</td>
<td>980</td>
<td>75</td>
</tr>
<tr>
<td>13/1200</td>
<td>997</td>
<td>45</td>
</tr>
<tr>
<td>14/1200</td>
<td>981</td>
<td>75</td>
</tr>
<tr>
<td>15/6000</td>
<td>972</td>
<td>90</td>
</tr>
<tr>
<td>16/0000</td>
<td>1000</td>
<td>45</td>
</tr>
</tbody>
</table>

1. From each table, convert the wind speeds of Hurricane Fran (1996) and Hurricane Carlos (2009) from knots, which are in nautical miles per hour (kt), to miles, or miles per hour (mph) (1 kt ≈ 1.15 mph). Then make a new table for each hurricane that includes pressure (in mb) and wind speed (in mph).

2. Using the data in the two tables, make a scatter plot for each hurricane (making the x-axis pressure and the y-axis wind speed, in mph). The data should form a linear pattern. Find the equation and slope for the line of best fit for each graph. Explain why the equation and slope mean.

3. Use one of your mathematical models to find the wind speeds (in mph) based on each of these pressures: 900, 1020, and 1070. Discuss the reasonableness of your results.

Extension: The Saffir-Simpson Hurricane Wind Scale (http://www.nhc.noaa.gov/shwsh.htm) uses top wind speed to categorize hurricane level. Determine the categories of Hurricane Fran and Hurricane Carlos. Explain why you chose that category level.

Jan A. Yow, jyow@sc.edu, is a National Board Certified teacher and mathematics education professor at the University of South Carolina. She is interested high-quality mathematics instruction for all students through mathematics teacher leadership.

The solutions are online with "Math for Real" at www.nctm.org/mtms...
Table 1: Names for tropical storms and hurricanes over the Atlantic-Caribbean basin

<table>
<thead>
<tr>
<th>Year</th>
<th>Name</th>
<th>Year</th>
<th>Name</th>
<th>Year</th>
<th>Name</th>
<th>Year</th>
<th>Name</th>
<th>Year</th>
<th>Name</th>
<th>Year</th>
<th>Name</th>
</tr>
</thead>
</table>

Note: Underlined names indicate tropical storms and hurricanes that occurred that year.

502  MATHEMATICS TEACHING IN THE MIDDLE SCHOOL  ●  Vol. 13, No. 9, May 2008
Paper Airplanes

Length of activity (based on 50-minute periods): 3-4 class periods

Objectives:
- Students will test their paper airplane making skills as they discover what variables affect the flight distance of their unique planes.
- Discovery of changes to mean, median, and mode as variables are changed.

Leading Question(s):
- What makes a paper airplane go fast? Far? Straight?

Materials Needed:
- Various weights of paper (enough for 1 plane each)
- Masking tape for floor
- Handout with questions
- A long hallway
- Tape measure

Integrated Skills Covered:
- Creativity
- Critical thinking
- Collaboration
- Problem Solving
- Communication
- Persistence
- Resilience to Failure

Mathematics SOL Strand Covered:
- 8.13 Probability and Statistics
- 8.17 Patterns, Functions, and Algebra

Lesson (see also attachments/handouts):

Students’ Responsibility
- In groups of 3-4, students brainstorm about the variables that affect the flight of a paper airplane.
- How can you change those variables when building a plane?
- On chart paper, or scratch paper, draw a design and list the variables.
- Build unique planes, and get ready to fly!
- Once in the hall, leave your plane until everyone has gone – this makes a real-life scatter plot!
- Take measurements carefully, and record your plane’s distances.

Teacher’s Responsibility

Day 1
- Talk with students about building a paper airplane.
- Break students into groups of 3-4.
- As a class, go over variables (lift, weight, drag, thrust).
- Allow students to choose their paper for their unique plane build (any weight, any size, any style), and begin building. Test flights may happen in the classroom depending on space/time.

Day 2
- Be sure the hallway is setup with a “runway.”
- Once each student has completed a unique plane, bring the class down to the hall for 1 test flight each, and 1 recorded flight.
- Collect the length of the flight, as well as how far off the “landing strip” the plane landed.
- What affected each of the measurements? The mean, median, mode, range? What could you do, as a class, to change those?
- Write down your prediction for how those measurements will change if everyone builds an identical rocket plane.

- Compare the data from both planes.
- Was your prediction correct?

• Once all data is collected, head back to classroom. If time allows, record all data on board for students to copy.
• Each student should find the mean, median, mode, and range of the class data and record it on their chart.

Day 3
• As a class, talk about the data collected.
• Have students answer questions.
• Show students the rocket plane design which they will be building and flying.
• Have students write down a prediction for what will change – mean, median, mode, range – and how they will change.
• Build rocket planes, and head down to runway if time allows.
• Collect data as before and record class data on the board.

Day 4
• Compare the rocket plane data to the unique plane data.
• Talk as a class about how it changed.
• Were their predictions correct?

Optional
Using graphing calculators, have students input the class data and graph a box-and-whiskers plot for both days to see how they compare.

Flying Paper Airplanes and Collecting Data

1. Go into the hallway in groups of three or four and fly your paper airplane.

2. When all the paper airplanes in your group have flown, take two measurements, in inches:
   - The first measurement: How far down the landing strip your plane flew.
   - The second measurement: How far off of the landing strip your plane landed (measure in a straight line perpendicular from the landing strip).

3. Since we are interested in the flight accuracy of your paper airplane, the distance away from the landing strip should be subtracted from the distance down the landing strip. Please record your flight data, and leave your paper airplane where it landed in the hallway.

4. You have permission to fly your paper airplane a second time if you have a negative number for your flight data. This could occur if you subtract your two measures and the distance off the landing strip is greater than the distance down the landing strip. If your plane lands behind you or flies a short distance down the landing strip and veers off quickly to the right or left, a negative number could also result.

5. Once you have collected your flight data, return to the classroom and record your data on the whiteboard.
Assessment Options:
- Have students record their class data, and show work for finding mean, median, mode, range
- Keep a mini-journal about their predictions and findings of each group of planes
- Informal conversations as a class

Variables to consider:

Data Collection Table Example:
Proofs in Grade 8

Length of activity (based on 50-minute periods): 1 day

Objectives:
- Allow students to put their ideas into words, and think critically about a problem at hand.

Leading Question(s):
- Is there an error in your formula sheet?

Integrated Skills Covered:
- Critical Thinking
- Creativity
- Problem Solving
- Communication
- Collaboration
- Resilience to Failure
- Persistence

Materials Needed:
- Grade 8 formula sheet

Mathematics SOL Strand Covered:
- 8.7 Measurement
- 8.10 Geometry

Lesson (see also attachments/handouts):

<table>
<thead>
<tr>
<th>Students’ Responsibility</th>
<th>Teacher’s Responsibility</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Begin with brainstorming about what could possibly be wrong with using the formula for a pyramid given on your formula sheet for all pyramids.</td>
<td>• This lesson should come after students are familiar with both the surface area of pyramids, and the Pythagorean Theorem.</td>
</tr>
<tr>
<td>• Talk with your neighbors or partner about your ideas.</td>
<td>• In the Grade 8 SOL formula sheet, there is an error in assuming that the formula for the surface area of any pyramid can be summed up with one formula. The formula given will only work for regular based pyramids.</td>
</tr>
<tr>
<td>• Begin to form a plan of action to discover what is wrong.</td>
<td>• If the side lengths are different, the slant height will differ, and so changing the surface area of the object.</td>
</tr>
<tr>
<td>• Once you have decided what the problem is, you must explain it in a way that others can understand.</td>
<td>• Without telling students all of this information, ask them to inspect the formula for the surface area of a pyramid carefully.</td>
</tr>
<tr>
<td>• You do not have to use formal math language, but be sure your argument makes sense to the reader! Showing examples may be helpful.</td>
<td>• Explain to them that there is, indeed, an error in thinking that this formula will work for all pyramids, and ask them to prove why.</td>
</tr>
<tr>
<td>• Do NOT give up if you start down the wrong path – your teacher is here to guide you, not give you answers.</td>
<td>• Be careful to not give them much information in the beginning – this is a lesson in creativity and persistence!</td>
</tr>
</tbody>
</table>
• You may choose to have students work in pairs or individually.
• You may need to give them the hint that the Pythagorean Theorem will come in handy in this proof.
• Do not force students to use “math talk” when working through this problem, or others like this. Allow them to grasp the concept in their own language first, and be creative in their explanation.

Assessment Options:
• Students may present their proof to the class
• Given a regular based pyramid, and an irregular based pyramid, show with numbers how the two surface areas (especially slant heights) differ

Formula given on SOL formula sheet (need S.A. only):

![Pyramid Diagram](image)

\[ V = \frac{1}{3} Bh \]
\[ S.A. = \frac{1}{2} lP + B \]

Other topics useful for proofs in grade 8:
• Why is a triangle inscribed in a semi-circle always right?
• Vertical angles are always congruent.
• Why do only regular triangles, squares, and hexagons create a regular tessellation?  
  **Extension:** Show why there are only 8 combinations of shapes that make semi-regular tessellations.
• Given a pyramid and a prism with equal base areas and heights, show why the volume of the pyramid is \( \frac{1}{3} \) that of the prism.
Sierpinski Tetrahedron

Length of activity (based on 50-minute periods): Approximately 1 week. Time will depend on number of students participating.

Objectives:
- Students will work together to build a piece of mathematical art; a Sierpinski tetrahedron.

Leading Question(s):
- What do mathematics and art have in common?

Materials Needed:
- 67-weight card stock
- Templates
- Tape
- Elmer’s glue
- Scissors

Lesson (see also attachments/handouts):

<table>
<thead>
<tr>
<th>Students’ Responsibility</th>
<th>Teacher’s Responsibility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Work together to build the Sierpinski Tetrahedron.</td>
<td>Discuss fractals, and their special properties.</td>
</tr>
<tr>
<td>This requires patience, and communication, especially in the final stages.</td>
<td>Talk and show real-life fractals, i.e. broccoli, romanesco broccoli, coastlines, CGI mountains.</td>
</tr>
<tr>
<td></td>
<td>If time allows, have students create their own fractal drawing (see attached).</td>
</tr>
<tr>
<td></td>
<td>Show students a picture of what they will be building.</td>
</tr>
<tr>
<td></td>
<td>Follow attached directions for construction of the Sierpinski Tetrahedron.</td>
</tr>
<tr>
<td></td>
<td>Students may be asked to calculate such things as area, perimeter, and how they change as the fractal grows (this could be surprising!)</td>
</tr>
</tbody>
</table>

Assessment Options:
- Completion of the final product
- Calculation of perimeters and areas, and description of how they change as the fractal grows

Integrated Skills Covered:
- Creativity
- Problem solving
- Communication
- Collaboration
- Persistence
- Resilience to failure

Mathematics SOL Strand Covered:
- 8.7 Measurement, Problem solving

Lesson adapted from Paul Kelley, Anoka High School
Stage 5 Sierpinski Tetrahedron:

Built by the geometry and algebra classes of Ana Teller and Trish Kridler, Auburn Middle School, 2013
Hand-Drawn Fractal Project

Due:

Your fractal must show:
1. Evidence of self-similarity
2. An iterative process

Other notes:
1. You must go to at least stage three, and you could go as high as stage ten or eleven or more, depending on the complexity of your fractal.
2. If stage n is your final product, I must see a rough draft of all stages from stage 0 to stage n-1. Feel free to put this on the back of your sheet.
3. You will receive extra credit for correctly computing the dimension of your fractal.

Hints:
1. Color often looks nice, especially when using contrasting colors to emphasize different aspects of your fractal.
2. Be creative.
Sierpinski Pyramid

To build a Sierpinski Pyramid, follow these steps:

1. Have enough copies made of the pyramid template to construct the desired size of pyramid. See the list below for the number of templates needed for each stage. We use 67-pound paper. Be sure to copy more templates than you'll need, as some will get crushed, others will be cut improperly, etc.

<table>
<thead>
<tr>
<th>Stage</th>
<th>Number of Templates</th>
<th>Approximate Height</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>4 inches</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>8 inches</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
<td>16 inches</td>
</tr>
<tr>
<td>3</td>
<td>64</td>
<td>32 inches</td>
</tr>
<tr>
<td>4</td>
<td>256</td>
<td>64 inches</td>
</tr>
<tr>
<td>5</td>
<td>1024</td>
<td>128 inches</td>
</tr>
</tbody>
</table>

2. Cut each template. NOTE: It seems most efficient to have the students cut out the templates the night before starting the project. Be sure to have them save the scraps, as they will need the scraps later, when putting together the various stages.

3. Fold each template to form a small, "Stage 0" pyramid. Tape the edges together.

4. Put four of these together to form a Stage 1 pyramid. Attach the four pyramids together at their vertices, using glue and a small, rectangular brace cut from the scraps of the templates.

5. Put four of the stage 1 pyramids together to form a stage 2 pyramid.

6. At this point, you might have to move your materials to the place where the completed pyramid will eventually reside, as going through one more stage will make a pyramid too big to fit through many doorways.

7. Continue the process described until you reach the desired size for your Sierpinski pyramid. A stage 5 pyramid will have as its base an equilateral triangle that is approximately 140 inches per side.

Materials needed:

Pyramid templates
Tape (approximately ten rolls)
Scissors (approximately 10-15)
Glue (one bottle per group of three students)

Before starting the project, go through in detail with the students exactly what it is that you're doing, and the step-by-step procedure. This will help them to see the "big picture."
Toothpick Patterns

Length of activity (based on 50-minute periods): 1 day

Objectives:
- Using manipulatives, students will come up with a geometric pattern, and a function to determine various attributes of the toothpick squares.

Materials Needed:
- Toothpicks
- Worksheet questions

Integrated Skills Covered:
- Creativity
- Problem solving
- Collaboration
- Communication

Mathematics SOL Strand Covered:
- 8.14 Patterns, Functions, and Algebra

Lesson (see also attachments/handouts):

<table>
<thead>
<tr>
<th>Students’ Responsibility</th>
<th>Teacher’s Responsibility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carefully build the first 3 patterns given.</td>
<td>Talk with students about patterns, functions, and representing the two with tables, graphs, and words.</td>
</tr>
<tr>
<td>Brainstorm with your partner about how you would like to represent your pattern.</td>
<td>Pair students together.</td>
</tr>
<tr>
<td>Using your method of choice, see if you can predict how many toothpicks will be used in the perimeter, and total figure, for the various stages.</td>
<td>Distribute one worksheet per pair, and toothpicks.</td>
</tr>
<tr>
<td></td>
<td>Have students build, and work through the problems at their own pace.</td>
</tr>
</tbody>
</table>

**NOTE:** This lesson could be modified to work with linking cubes, flat shapes, or any other shape that is readily available. Area could also be introduced if using square blocks.

Assessment Options:
- Collect questions from pairs
- Talk as a class about how to represent a pattern, and predict using that representation (function)
1. With a partner, build ONLY the following three patterns out of your toothpicks.

2. Use the pattern above to determine the perimeter of staircase 5.

3. Use the pattern above to determine the total number of toothpicks in staircase 5.

4. Explain how you can determine the perimeter of any figure like the pattern above.

5. Explain how you can determine the total number of toothpicks in any figure like the pattern above.

6. Using your idea in problem 4 & 5, determine the perimeter and number of toothpicks in staircase 100.