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# Problem Posing in a Fourth Grade Classroom: Analyzing the Impact of Weekly Practice

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Abstract

As mathematics education moves away from rote memorization and procedural methods, conceptual approaches such as problem posing are gaining support. Problem posing refers to the generation of new mathematical problems and the reformulation of given ones (Chapman, 2012). It requires higher order thinking skills and has become a recognized means of developing mathematical thinking and creativity in students of all ages (Koichu & Kontorovich, 2013). This mixed methods, action research study explores the impact of a weekly problem-posing instructional intervention on the mathematical problem posing and problem solving abilities of fourth grade students. The problems students posed in response to artifact prompts (restaurant menus, grocery circulars etc.) are evaluated both quantitatively and qualitatively. Analysis focuses on how students' problem-posing performance developed over the course of the study, and on the potential connections between problem posing and other achievement variables.

## Introduction

Over the past few decades the emphasis of mathematics education has shifted away from rote memorization and procedural knowledge, towards a more conceptual understanding of the mathematical world. Through this shift, problem solving has gained a lot of attention and has been incorporated into many mathematics standards and curricula. Although not as widely publicized, especially in the United States, research on another aspect of conceptual mathematics – problem posing – has also been quietly accumulating. This relatively new facet of mathematics education has been gaining ground and is now “strongly recommended” by The National Council of Teachers of Mathematics’ (NCTM) (Silver, 2013). Throughout the world of mathematics education, problem posing is becoming a recognized means of developing both mathematical thinking and creativity in students of all ages (Koichu & Kontorovich, 2013).

A growing body of literature has begun to document and explore the many ways that problem posing can be utilized in the mathematics classroom. Although this increase in research has brought about a broadening and blurring of definitions, problem posing continues to refer to both the generation new problems and the reformulation of given ones (Chapman, 2012). Problem posing requires higher order thinking skills and is in some ways, even more rigorous than problem solving (Bonotto, 2013). The literature discusses a wide assortment of uses and contexts for problem posing. While the prior research provides valuable insight into how to effectively implement problem posing, the various types of prompts that can be used, and the various ways student work can be analyzed, the literature is not as thorough in its discussion instruction in problem posing.

# Literature Review

## Benefits and Rationales

The rationales for incorporating problem posing into the mathematics curriculum include practical as well as mathematical benefits. As Silver (2013) explains, problem posing can be a goal in and of itself, or it can be a means to other goals. Thus for some researchers and educators problem posing is viewed as a component of a larger problem-solving framework. In their article on mathematical reflection, Krulik and Rudnick (1994), discuss problem posing as one of several tasks students can and should engage in after solving a problem. From this perspective problem posing is means for extension and reflection during mathematical problem solving.

Other studies view problem posing as a means of strengthening and improving students' mathematical motivation and engagement. Whitin (2006) argues that problem-posing tasks foster a sense of ownership and a "spirit of inquisitiveness" (p. 17). Problem posing is also believed to improve critical and creative thinking skills (Krulik & Rudnick, 1994; Bonotto, 2013) and help students become "braver" problem solvers (Barlow, 2007, p. 255). These intangible skills are not directly tied to mathematics but certainly contribute to higher performance.

The literature also emphasizes that instruction in problem posing is necessary to sufficiently prepare students for success in the 21st Century. Bonotto (2013) argues that the word problems students are typically given in school are far too simplistic and contrived. They do not require true problem-solving skills since students can simply look for syntactic clues and apply procedural operations (p. 37). In the real world, she argues, mathematical problem are rarely so cut and dry; part of the challenge of true problem solving is first determining the nature of the problem itself.

In addition to providing practical preparation for the future, problem-posing tasks are also a practical differentiation tool for teachers. With ever increasing levels of diversity, teachers are faced with the daunting task of providing learning opportunities that are challenging and meaningful to a wide range of learners (Koichu & Kontorovich, 2013). Having students generate their own problems based on a given prompt allows for multiple points of entry based on readiness and proficiency. While students who are behind in mathematics can pose simple, one-step problems, more advanced students can have the opportunity to pose complex problems. These students can create multi-step problems or incorporate more challenging mathematics concepts such as ratios, algebra or hypothetical changes to the constraints (Leung, 2013). This built-in potential for differentiation makes problem posing a practical tool for teaching diverse groups of students.

**Suggestions for Implementation**

The literature offers a variety of suggestions for effectively implementing problem posing in the classroom. In her article on the responsibilities of a teacher in a harmonic cycle of problem solving and posing, Chang (2007) emphasizes the importance of teaching problem solving prior to teaching problem posing. She explains that young children need to learn how to solve problems posed by their teacher before they are asked to generate their own. This approach to problem posing emphasizes the structured role of the teacher in choosing the objective, planning the experience, and selecting the materials (Chang, 2007). Whitin (2006) on the other hand, suggests a more open-ended, exploratory approach to problem posing. He suggests posing problems with multiple solutions and using students' responses as a jumping-off-point for student-centered exploration of number relationships and mathematical generalizations.

Regardless of perspective on the order of teaching problem posing and problem solving, there seems to be consensus throughout the literature about the importance of establishing clear expectations for problem-posing tasks. Think alouds can be a useful tool for modeling how to go about posing mathematical problems based on given information (Whitin, 2006). It is also important to ensure that students understand the difference between a problem and a statement before they are asked to pose their own problems (Barlow, 2007). If problem posing is not adequately explained and modeled, it is likely to be frustrating und unfruitful. Posing mathematical problems is a new experience for most students and consequently requires significant scaffolding, especially at the beginning.

In addition to thorough modeling, there are also several strategies that can make problem-posing tasks more successful. In her study on problem posing in Title I schools, Barlow (2007) finds that when students are given a number (e.g. 12) as a solution they tend to generate only a number sentence (e.g. What is 3 x 4?). But when the solution is given in specific units (i.e 12 cookies) students are more likely to generate a full story problem. Additionally, having students solve one another's problems aids them in becoming critical analyzers (Bonotto, 2013). Often the problems students pose make sense to them but are not clear enough for a peer to solve. Having students revise and peer edit their problems so that other students can solve them brings problem posing tasks to an even more rigorous level (Leung, 2013).

**Problem-Posing Prompts**

Although problem-posing tasks come in many varieties, they all typically ask students to generate one or more problems based on some piece of given information or prompt. Based on the literature there are four main categories of prompts that can be used in problem posing tasks.

The majority of problem posing studies involve context prompts. These prompts provide students with a real-world context for the problems they are to pose. For the problem-posing task discussed by Silver and Cai (1996), students are given a structured list of facts (Arturo drove 80 miles more than Elliot, Elliot drove twice as many miles as Jerome etc.) and are asked to generate three *different* problems based on the information. Although the specific context (a road trip) is prescribed, the prompt provides the building blocks for a variety of mathematical questions. Similarly, the Billiard Task, a well-known problem-posing prompt, provides students with a diagram of a billiard table and the specific path of a ball (Koichu & Kontorovich, 2013). Students are asked to use the diagram to create a series of mathematically interesting problems. Again, although the specific context is non-negotiable, the mathematical options are unlimited.

Context prompts can also be less structured. In a study conducted in Italy students were given ‘cultural artifacts’ (restaurant menus, amusement park pricing brochures, coupon booklets etc.) as prompts for the creation of mathematical problems (Bonotto, 2013). With this approach, students were provided with an abundance of numbers and possibilities and part of the challenge was deciding what to focus on. Students had to generate the scenario as well as the problems. Similarly the informal contexts used in English’s (1998) study, involved providing students with a photograph or piece of literature as the context for their posed problems. Students were provided with more information than they needed and had to decide what to focus on.

The second variety of problem-posing prompts discussed in the literature are constraint prompts. Rather than dictating the specific real-world context (a road trip, a billiard game etc.) these prompts dictate the mathematical structure. The tasks used in Chapman’s (2012) study of prospective teachers approaches to problem posing, involved a list of prompts that each started with “Create a word problem that is… [for students in a certain grade level; related to the concept of multiplication; similar to this given problem etc.].” The participants could choose any real-world context, but their problem had to meet the specific mathematical criteria. Although Chapman’s (2012) study was conducted with adult participants, a similar type of constraint prompt could easily be employed with elementary students. The problem-posing tasks discussed by Krulik and Rudnick (1994) also use a form of constraint prompts. After solving a given problem, students are asked to pose "what if" questions in order to explore the cause and effect relationships that exist between givens and results. (e. g. Suppose the farmer had 50 cows instead of 30. How would that change his weekly milk production?). The problem posing is constrained by the mathematical structure (and to some extent the context) of the original problem.

A third variety of problem-posing prompts involve providing students with a number sentence around which to create their problems. Whitin's (2006) approach to problem posing involves wonder, exploration and "what if" questions. Students are asked to create a list of observations about a given number sentences (such as 5 x 4 = 20) and subsequently a list of questions (such as: Is an odd number times an even number always an even number?). This approach is very open-ended and is aimed at helping students develop a more critical, inquisitive approach to mathematics (Whitin, 2006). English (1998) also uses number sentences as prompts for problem posing but, rather than generating a series of exploratory mathematics questions, the students were simply asked to create three story problems. With the number sentence dictated students have somewhat limited options mathematically but unlimited contextual possibilities.

The final variety of problem-posing prompts discussed in the literature involves providing an answer and having students generate a problem. The tasks used in Barlow's (2007) study use the structure of "The answer is…. [45 red cars, 15 snowmen, 20 cookies etc.]." Students are asked to create a word problem that would produce the given answer. In this case students have unlimited mathematical possibilities and only a mild contextual constraint.

These four types of problem-posing prompts – context, constraints, number sentences and answers – provide varying levels of challenge and possibility. Rather than seeing these prompts as having different values, it is more useful to view them as different tools for different situations. Depending on the age group and the objective of the task different prompt structures may be more or less appropriate.

**Analyzing Student Work**

Mathematical problem-posing tasks yield rich insights into students' mathematical thinking and can be analyzed in a variety of ways. The problems students pose reveal a lot about their thinking. The analyses outlined in the literature generally fall into two categories: those that focus on the product (the problem posed) and those that focus on the problem-posing process (how the problem is generated).

The simplest level of product analysis focuses on whether or not the problems students pose are mathematically sound and solvable. In her study on implementing problem posing in elementary classrooms in Taiwan, Leung (2013) categorized students posed problems using a 1-5 coding system based on whether or not the problem was math-related, and whether or not it made sense and provided sufficient information. This basic level of analysis separates mathematical from non-mathematical problems and solvable from unsolvable problems. With this approach any solvable, mathematical problem is considered sufficient and acceptable.

A more in depth type of product analysis involves categorizing students' posed problems according to their structure and/or type. English (1998) analyzes the structure of the addition and subtraction problems that third grade students posed. Basic part-part-whole problems with the whole unknown or one part unknown are over-represented in mathematics curricula and he was interested in investigation whether or not this trend would extend to the problems students pose. He categorized the posed problems by their form – whether basic (e.g. Jessie has 3 marbles, Tom has 7. How many do they have altogether?), comparison (e.g. Jessie has 3 marbles, Tom has 4 more than Jessie. How many do they have altogether?), or equalize (e.g. Tom has 7 marbles, Jessie has 3. How many more does Jessie need in order to have the same number as Tom?). By categorizing students' problems by to structure, English (1998) heightened the criteria for success by moving beyond simply determining whether or not the problem was mathematical. The quantitative portion of Van Harpen and Presmeg's (2013) research also categorized the problems students posed, but in this case they were categorized by problem type - arithmetic, algebra, probability etc. For their study, problem type served as a proxy for complexity. Posing an algebraic problem was considered more advanced than posing an arithmetic problem.

The second group of studies takes a step back from the finished product (the posed problem) and instead examines the process of problem posing. In their study of how college students generate interesting mathematics problems from the Billiard Task prompt, Koichu and Kontorovich (2013) analyze participants' problem-posing strategies and their reasoning approaches. Participants were asked to write about their approach to developing problems. The study found that participants went through similar stages as they became acquainted with the prompt and attempted to create mathematically interesting problems. Although they initially relied on prototypical problem structures, participants tried to make their problems unique in some way and tried to cover up the problem creation process in the final product (Koichu & Kontorovich, 2013). Similarly, Chapman (2012) analyzed the sense making processes of teachers as they attempted to pose problems. She found that there were five perspectives that participants subscribed to as they generated their problems: paradigmatic (creating problem for a general audience), objectivist (working backwards from a specific math fact or concept they wanted to include), phenomenological (creating problemss with opportunities for personal relevance to the solver), humanistic (personalized to solver), and utilitarian (creating problems based on their potential contribution to student learning). Examining the process of problem posing, acknowledges that often there is more to the final product than we can see on the surface. By having participants share and reflect on their process, the researcher gains a clearer understanding of what the problem poser was thinking as they created the problem

**Linking to Other Variables**

In addition to studying problem posing itself, several studies also attempt to determine whether or not problem posing is correlated with other achievement variables. Bonotto (2013) explores the relationship between problem posing and creativity in her study of fifth grade students' problem posing. Creativity is a very abstract concept and can be difficult to measure. Several studies have defined the core dimensions of creativity as fluency, flexibility and originality and thus, for her study, Bonotto, (2013) analyzes the problems students posed according to those categories. She compared the results of students from two different schools, and found that the students from the second school scored higher on all the measures of creativity. Since the second school had higher average academic achievement, further research would be necessary to determine the nature of the correlation between academic achievement, creativity and problem posing.

Instead of creativity, English (1998) examines the relationship between problem solving and number sense, and problem-posing ability. She created four categories (shown in Table 1) and made sure each group was represented in her sample. Half of the students from each achievement category participated in an eight week problem-posing program. On the pre-test

Table 1

*Matrix of ability groups used in English's (1998) study*

| | **Strong Problem Solving** | **Weak Problem Solving** |
|---|---|---|
| **Strong Number Sense** | SN/SP (20) | SN/WP (20) |
| **Weak Number Sense** | WN/SP (14) | WN/WP (13) |

there was no significant variation in problem posing between the achievement categories. Children from all groups showed a strong preference for the basic change/part-part-whole problems. Interestingly, participation in the problem-posing program did not enhance student problem-posing competence as much as one might expect. Even with additional instruction and encouragement most students struggled to see beyond the basic problem structure and continued to pose simple part-part-whole problems. Overall neither achievement group nor participation in the program seemed to play a very decisive role in students' problem posing. Since the study involved so many variables and a small sample size, the relationship between number sense, problem solving and problem posing also requires additional documentation.

Silver and Cai (1996) also explore the relationship between problem posing and problem solving but in a simpler way. They grouped students into a "Hi" and a "Lo" group based on their scores on a problem-solving test and then compared the problems the two groups posed. The findings indicate that the Hi group posed more mathematical problems. The majority of the non-mathematical problems were posed by students in the Lo group. Although this seems to suggest that there is indeed a correlation between problem solving and problem posing, with this limited data it is impossible to determine whether or not there is actually a latent variable (such as IQ or academic achievement) that is actually influencing both problem solving and problem posing.

In addition to creativity and problem-solving ability, Van Harpen and Presmeg (2013) examine the relationship between Chinese and American high school students' mathematical content knowledge and their problem-posing ability. Students were given both a content knowledge test and a series of problem-posing prompts. Within group analysis found no significant correlation between the variables but further research of this topic would be beneficial since the sample size for Van Harpen and Presmeg's (2013) study was fairly small.

Although these studies provide useful information regarding how to go about measuring and comparing complex variables, ultimately most of their findings are only hazy at best. It seems that more research is necessary in order to clearly establish if and how problem posing is related to other variables.

**The Present Study**

The academic literature on problem posing provides a rich discussion of contexts and uses for problem posing in the classroom. The studies discussed above provide useful insight into the benefits of using problem posing, and the varying impacts of different types of prompts and analysis. Most of the literature on problem posing focuses on students' problem-posing ability as a snapshot in time, rather than a developing skill. Only English's (1998) study looks at the effect of a problem-posing program. Since her study also looks at a variety of other achievement variables it is difficult to draw conclusions based on her findings.

Thus the present study seeks to expand upon existing literature by exploring the impact of a weekly instructional intervention on the problem-posing performance of fourth grade students. With the academic literature as a guide this study addresses the following research questions:

1. How do weekly problem-posing workshop sessions impact the performance of high and low ability problem posers?
2. To what extent is there a relationship between students' problem-posing and problem-solving abilities?
3. How do weekly problem-posing workshop sessions impact the complexity and creativity of the problems students generate?
4. How do weekly problem posing workshop sessions impact the thought process associated with problem generation?

# Methodology

In order to explore the impact of a problem-posing instructional intervention, I conducted a mixed methods, action research study with the fourth grade class in which I was student teaching. Quantitative analysis was used to compare mean scores within and between groups and to explore potential correlations. Qualitative analysis was used to evaluate students' approaches to problem posing and how the problems they posed changed over time. The combination of statistical data and descriptive, narratives provides a well-rounded picture of the impact of problem-posing instruction at the elementary school level.

## Setting and participants

The study was conducted in a fourth grade class at suburban elementary school about an hour outside of the Washington D.C. metropolitan area. This site was selected because it is where I was completing my student teaching internship. The school has an enrollment of 539 students – 255 female and 284 male and receives targeted Title I assistance. About 33% of the student population receives free/reduced lunch. The racial composition of the school is summarized below.

Table 2

*Racial Composition of the School*

| Racial Category | Percentage |
|---|---|
| White | 59.1 |
| Black | 18.8 |
| Hispanic | 11 |
| Two or more races | 8.5 |
| Asian | 2 |

In addition to the demographic information above, it is also important to note that the school uses Investigations (https://investigations.terc.edu), a very conceptual, hands-on mathematics curriculum. As a result, students who have been at the school for several years, tend to have strong number sense and reasoning skills by the time they reach fourth grade. This is in stark contrast to many elementary schools where procedural approaches to mathematics pervade the curriculum, and large numbers of students lack conceptual foundations in mathematics

Participants included 18 fourth-grade students: 11 males and seven females. The majority of the participants were Caucasian; however, there were three African American students and one Hispanic student. Of the 18 participants, there were three English Language Learners (ELLs), two students with 504 plans, and six students in the gifted program. The students in the class enjoyed math, and they had already had some experience with writing their own story problems earlier in the year. This class provided a wonderful context for exploring the impact of using problem-posing tasks in the elementary classroom.

**Data Collection**

The impact of the instructional intervention (the weekly workshop sessions) was measured using a pre- and post-test, weekly problem-posing exit slips, problem-solving task cards, researcher observations, and exit interviews.

**Pre- and post-test.** Prior to the first workshop sessions, students were given a problem-posing pre-test to assess their ability to pose mathematical problems (Appendix A). The pre-test contained four different prompts that represented four of the prompt types discussed in the literature. All four prompts were based on a provided section of a simple restaurant menu but each was structured differently. The first prompt was a context problem similar to the prompts used by Silver & Cai (1996). Students were given a series of statements about the artifact (menu) and were asked to create a story problem related to those statements. The second prompt left the context wide open but dictated a mathematical constraint: the problem had to include multiplication. The third prompt was completely unrestricted: students could pose any problem they wanted as long as it included something from the provided menu. The final prompt asked students to improve upon their problem from prompt three to make it more detailed or complex.

In order to minimize any external influences, students were given only a very general introduction to the project or the pre-test. They completed the pre-test independently and their performance served as a baseline against which their post-test performance was compared. At the end of the study, students completed the same assessment as a post-test.

**Workshop sessions.** Students participated in four problem-posing workshop sessions during the course of the study. The sessions provided opportunities for students to practice posing problems and become more comfortable with the process. They also offered strategy instruction to help students move towards posing more complex and creative problems. Complex problems are those that involve multiple steps and/or more advanced mathematics (e.g. multiplication and division and even higher mathematics such as ratios, probability, percentages). Creative problems are those that introduce some combination of cognitive demand, novelty, and originality (Koichu & Kontorovich, 2013) by moving beyond the information provided in the prompt.

Each workshop session was designed to include: 1) a problem-posing warm up prompt, 2) discussion of a new skill, 3) partner practice with the new skill, and 4) an independently completed exit slip. Each week's session focused on a different skill associated with successful problem posing:

Week 1 – Choosing a focus; creating and solving a mathematics-related problem

Week 2 – Providing sufficient information; solving each other's posed problems

Week 3 – Creating complex problems, multiple steps and advanced problem structures

Week 4 – Creating creative problem, introducing new information beyond the prompt

At the end of each workshop session, students independently completed a single prompt exit slip (Appendix B). Each exit slip had a different artifact around which students were to pose their problem. The prompts on the exit slips were very open-ended and were comparable to Prompt 3 on the pre-test.

**Problem-solving task cards.** In addition to the problem-posing exit slips, students also completed four problem-solving tasks cards during the course of the study. Each task card included one single and one multi-step problem (Appendix C), and the difficulty level of the word problems remained constant throughout the duration of the study. The task cards served as a measure of students' problem-solving ability.

I intended to give the problem-solving task cards every Friday but due to snow days and other scheduling conflicts that did not happen exactly as planned. The final task card was actually given after the students had already taken the problem-posing post-test. Since the problem-solving task cards were pretty quick for the students to complete they were worked into the schedule wherever there was time.

**Process interviews.** In order to gain deeper insight into students' problem-posing thought processes, I conducted exit interviews with six students. Students were only interviewed if they had returned their parental consent form (Appendix D). All students were also read an assent letter and were given the option not to participate (Appendix E). The six selected students represented high, average and creative problem posers. Each interview was conducted in the hallway outside the classroom. I gave each student a couple minutes to look over the problems they had posed over the course of the study (pre- and post- tests as well as the exit slips) and then I asked them questions about how they came up with their problem, how they decided what to focus on and how they feel about the problem-posing tasks (Appendix F). In addition to the scripted questions, I used follow-up questions as necessary to elicit substantive answers. Interviews lasted approximately four minutes and were conducted during reading group time or bus dismissal. As agreed to in the parental consent form, interviews were audio recorded using an iPhone. Files were stored on a secure, private computer for the duration of the study and then deleted. These interviews provide valuable insight into the unique ways that elementary students approach the problem-posing process.

**Data Analysis**

The problems students posed on the pre- and post-test were evaluated in six categories using a researcher-developed rubric (Appendix G). The categories were drawn from the literature and included solvability, adherence to the prompt, complexity (number of steps), complexity (type of problem), creativity and solution. For the purpose of this study, creativity was defined as the introduction of new information above and beyond what was provided in the prompt. In order to receive full points in this category, students had to introduce new information in a way that was mathematically necessary for solving. The Complexity (Type) category assessed the type of mathematics required to solve the problems students pose. In order to score a four in this category, students had to use mathematics beyond the four operations with whole numbers (fractions, probability, etc.). I did not expect the fourth grade students to receive full credit in this category, but I chose to leave it on the rubric in order to leave room for students to surprise me. Realistically, I was looking to see students get threes in the Complexity (Type) category.

Each prompt was scored individually and averages (by prompt and by rubric category) were also calculated. This data was analyzed using descriptive statistics to determine growth at the individual, group and class level (Appendix H). The groups used for this study were: males, females, English Language Learners (ELLs), high problem posers (the five students with the highest scores on the pre-test) and low problem posers (the five students with the lowest scores on the pre-test). The problems posed on the post-test were compared to those posed on the pre-test using numerical comparisons *and* narrative descriptions.

The problems students generated on their weekly exit slips were evaluated using the same rubric as the pre- and post-tests (except the ‘adherence to the prompt’ category was not since it was not applicable). Each student’s growth was tracked over the four weeks of problem-posing workshop sessions, and group and class averages were also determined for each week. Average scores across all four exit slips were also determined for each student and each rubric category.

The problem-solving task cards were scored using the solution section of the scoring rubric. Each of the two questions was scored and the scores were averaged. The scores on each of the four task cards were also averaged to determine each student’s problem-solving average.

In order to analyze the student interviews, the audio recordings were transcribed. The interviews were then coded to identify patterns and common themes. Since each was asked the same interview questions, I organized the qualitative transcription data by question. Students’ responses and explanations were also compared to the problems they posed on the pre- and post-test and on the exit slips. Additionally, I wrote a memo of each interview to record details about the context of the interview, the student’s mood and any other noteworthy details.

The data collection and analysis procedures for this study are intended to provide a well-rounded understanding of students’ problem-posing abilities and their growth as a result of the instructional intervention. Because the sample size of this study is so small, results must be considered carefully and are not necessarily generalizable. Despite the limitations of this research, the multi-faceted nature of the study provides a rich array of data and offers useful insights into the impact of using problem-posing tasks in the elementary school classroom.

# Results

## Pre- and Post-Test

Overall, students' problem-posing skills improved during the study. There was a gain of 14 percentage points between the average pre-test score (58%) and post-test score (72%). Table 3 compares the pre- and post-test scores using several measures of central tendency.

Table 3

*Variation in measures of central tendency on the pre- and post-test*

| | **Pre-test** | **Post-test** |
|---|---|---|
| Mean Score | 52 | 78 |
| Range | 15 - 78 | 38 - 91 |
| Median Score | 65.5 | 74 |
| Mode Score | 68 | 74 |

The median score increased nearly 10 points, and the mode on the post-test was six percentage points higher than the mode on the pre-test. The highest score on the post-test was 13 points higher than the highest score on the pre-test, and the lowest score on the post-test was 23 points higher than the lowest score on the pre-test.

Figure 1 visually depicts the spread of scores on the pre- and post-test. The inner-quartile range on the post-test was significantly smaller than the range on the post-test (6 percentage points compared to 24 percentage points).

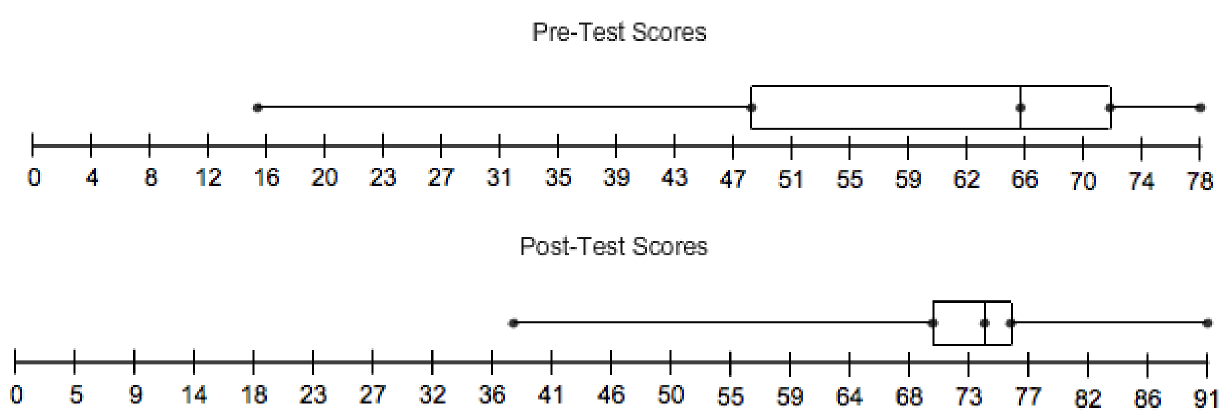


*Figure 1*. Box and whisker plots showing the spread of data on the pre- and post-test

The pre- and post-test scores were also disaggregated by group (Figure 2). In every group, the average post-test score was higher than the pre-test score. The graph also shows that each group's pre-and post-test scores were fairly similar to the overall class average. The males

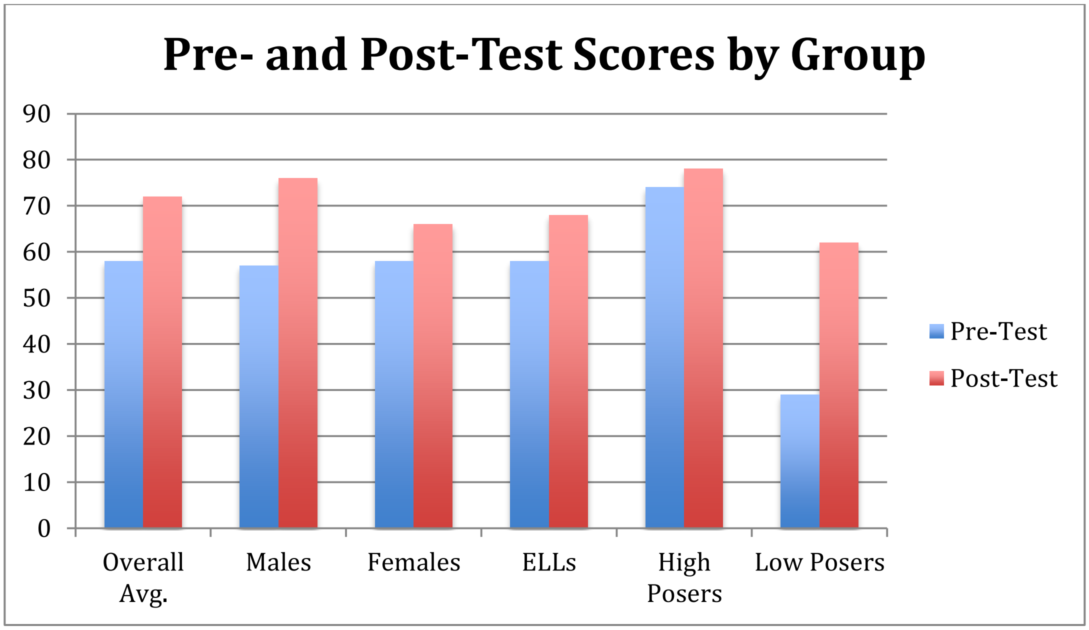


*Figure 2.* Bar graph showing the average pre- and post-test scores for each sub group

had a greater gain than the females (19 percentage points compared to eight percentage points). As might be expected, the high problem posers – those who scored the highest on the pre-test – had the least growth over the course of the study (4 percentage points), and the low problem posers – those who scored the lowest on the pre-test – had the most growth (33 percentage points). The average score for the ELL students increased 10 percentage points from the pre-test to the post-test

**Exit slips**

Over the course of the study, scores on the exit slips increased. The average scores on each set of the four exit slips are shown in Table 4. The scores on Exit Slip 3 and Exit Slip 4 were considerably higher than the scores on the first two sets of exit slips. The average score on

Table 4

*Average scores on exit slips*

| | Average Score |
|---|---|
| Exit Slip 1 | 64 |
| Exit Slip 2 | 63 |
| Exit Slip 3 | 79 |
| Exit Slip 4 | 76 |

Exit Slip 3 (79%) is 16 percentage points higher than the average score on Exit Slip 2 (63%). The average score on Exit Slip 4 (76%) is 12 percentage points higher than the average score on Exit Slip 1 (64%). This growth is comparable to the growth made from the pre-test to the post-test (14 percentage points).

**Research Question 1: High and Low Problem Posers**

Based on their pre-test scores, students were grouped by problem-posing ability. The five highest scoring students on the pre-test became my high problem-posing group, and the five lowest scoring students on the pre-test became the low group. Figure 3 shows the pre- and post-test scores for each of the students in the low problem-posing group. The Low Posers' pre-test scores ranged from 15% to 48% with a mean score of 29%.

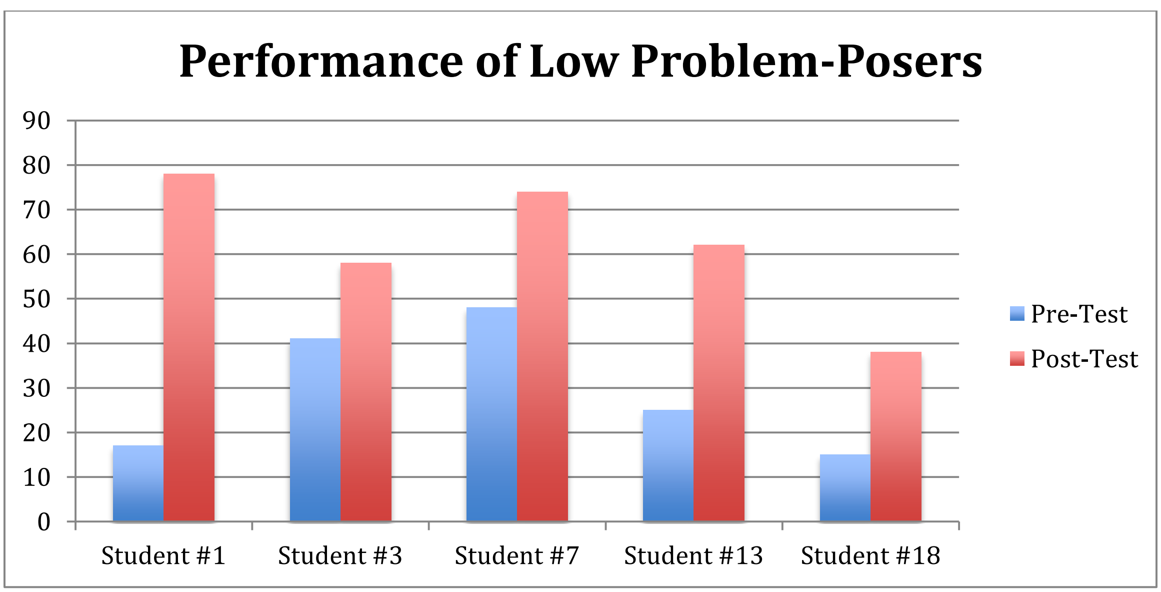


*Figure 3*. Bar graph showing the performance of low problem posers on the pre- and post-test

The students in the low problem-posing group showed varied growth throughout the course of the study. Student #1 made the most growth – 61 percentage points. He scored the fourth highest score in the whole class on the post-test. Student #3's post-test score was 17 percentage points higher than her pre-test score but was only a 58%. Student #7 had 26 percentage points of growth from the pre-test to the post-test.

On the exit slips Student #18 scored of 55%, 65%, 95% and 75%, but her scores on the pre- and post-test were much lower (15% and 38%). On both the pre- and post- test she only responded to two of the four prompts. She scored well on the prompts she did respond to. For example, on the post-test she wrote and correctly solved the following problem:

> [Janet] and Abby went to Sonny's. Abby got the large soft drink with a cheeseteak. Janet got a large soft drink with a hot dog. For desert Abby got 5 fried oreos and Janet got Funnel Cake. How much money did Abby's meal cost and How much money did Janet's meal cost?"

Student #13 had consistently low scores but did better on the post-test.

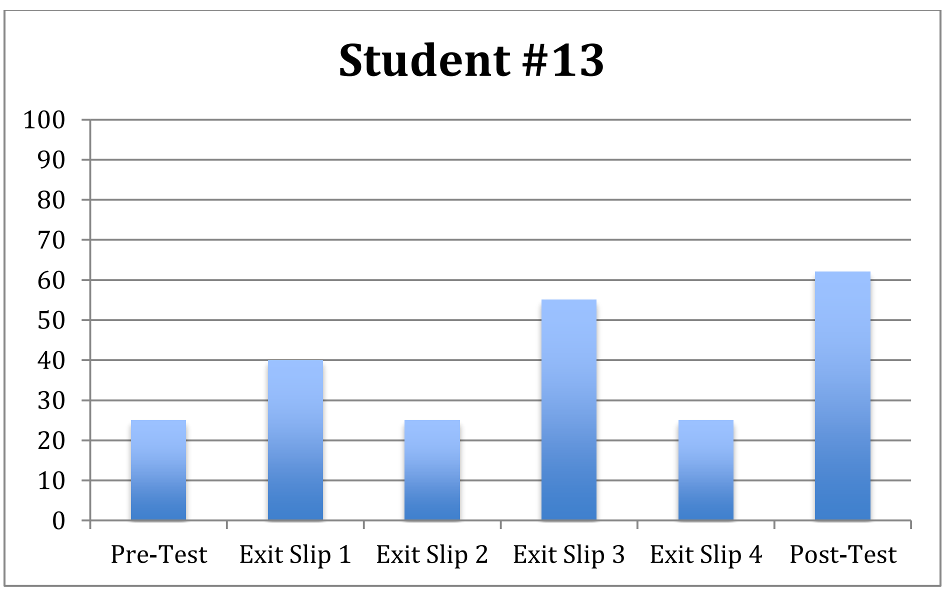


*Figure 4.* Student #13's scores on each component of the study

His story problem for Exit Slip 2 was:

Jack bought a W.C.S. [waffle cone sundae] how much did he pay?

This problem does not require any math; the answer is simply the price on the menu. Student #13 did not make very much progress over the course of the study. His score on Exit Slip 4 was the same as his pre-test score (see Figure 4). However, his post-test score was 37 percentage points higher than his pre-test score (62% compared to 25%). His problem for Prompt 2 on the post-test was:

Somebody bought two of each combo. Times two. So four of each. How much?

This problem scored considerably higher than his responses on the pre-test and exit slips.

The high problem posers scored much higher on the pre-test, but did not make as much growth over the course of the study. They had an average pre-test score of 74% (45 percentage

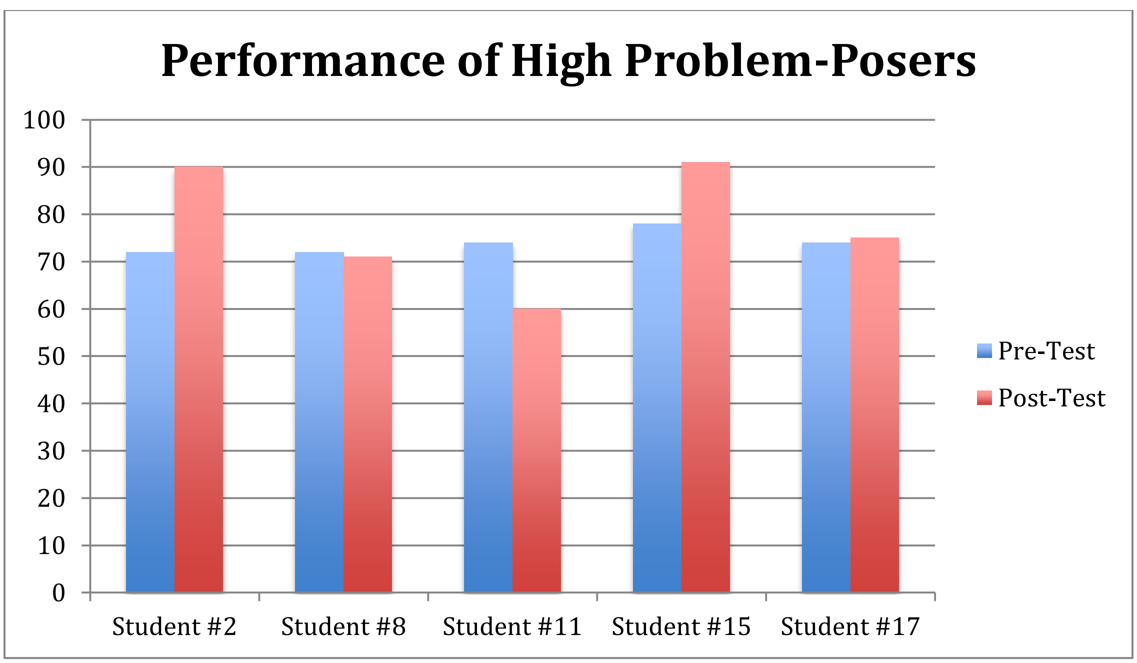


*Figure 5*. Bar graph showing the performance of high problem posers on the pre- and post-test

points higher than the low problem posers). As Figure 5 shows, this group did not have nearly as much growth as the low problem posers. In fact, two of the initially high problem posers (Student #8 and Student #11) had negative growth from the pre-test to the post-test. Student #17 had only nominal gains from the pre-test to the post-test.

**Research Question 2: Problem Solving versus Problem Posing**

Overall, students scored very well on the problem-solving task cards. Scores were fairly consistent across the four weeks, and the overall class average on the problem-solving task cards was 89%. Table 5 shows the average scores for each week. On the task cards for the first and third weeks, the class average was higher on the multi-step problems than the single step problems. But on the second and fourth sets of task cards, the average score on the single step problem was higher than the average score on the multi-step problem.

Table 5

*Average scores on problem-posing task cards by week.*

| **Week 1** | **Week 2** | **Week 3** | **Week 4** |
|---|---|---|---|
| 81 | 92 | 88 | 92 |

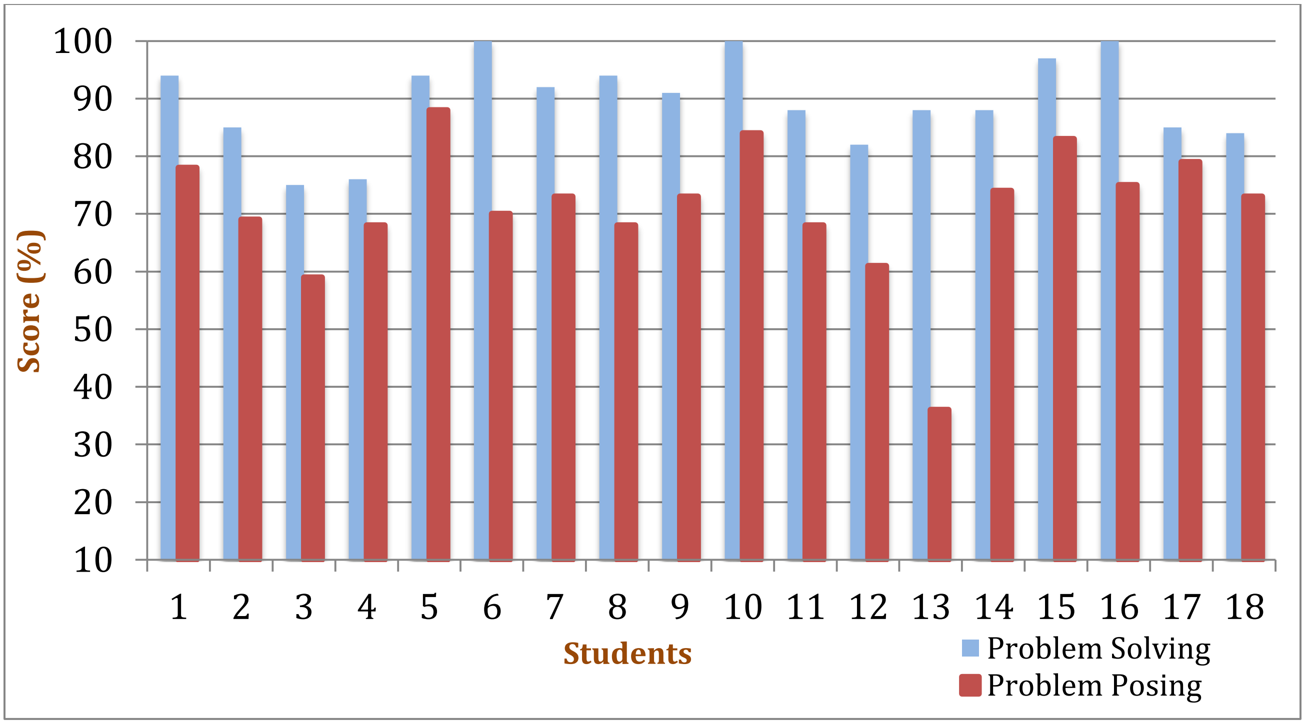


*Figure 6.* Bar graph showing each student's average problem-posing and problem-solving scores

As Figure 6 indicates, every student's average problem-solving score was higher than their average problem posing score. Despite the visual appearance of a correlation, there was not a statistically significant correlation between these two variables.

Table 6 shows the highest and lowest achieving students in both problem posing and problem solving. Of the high problem solvers, only one (Student #15) was also in the high

Table 6

*High and low problem solvers and problem posers*

| | **Problem Solving** | **Problem Posing** |
|---|---|---|
| **High** | Student 16 (100%) | Student 15 (78%) |
| | Student 15 (100%) | Student 17 (74% |
| | Student 10 (100%) | Student 11 (74%) |
| | Student 6 (97%) | Student 8 (72%) |
| | | Student 2 (72%) |
| **Low** | | |
| | Student 3 (75%) | Student 18 (15%) |
| | Student 4 (76%) | Student 1 (17%) |
| | Student 12 (82%) | Student 13 (25%) |

| | | |
|---|---|---|
| | Student 18 (84%) | Student 3 (41%) |
| | | Student 7 (48%) |

problem-posing group. Of the low problem solvers, only two students (Students # 3 and #18) were also in the low problem-posing group

**Research Question 3: Complexity and Creativity**

**Rubric category**. In order to evaluate the impact of weekly problem-posing practice on the complexity and creativity of the problems students posed, I analyzed the exit slip data by rubric category. Figure 7 shows the average score in each rubric category on each exit slip. The graph illustrates the variation in students' performance on the five different rubric categories. Across all four exit slips, students scored highest in the solvability category and the solution category (See Appendix G for a copy of the rubric).

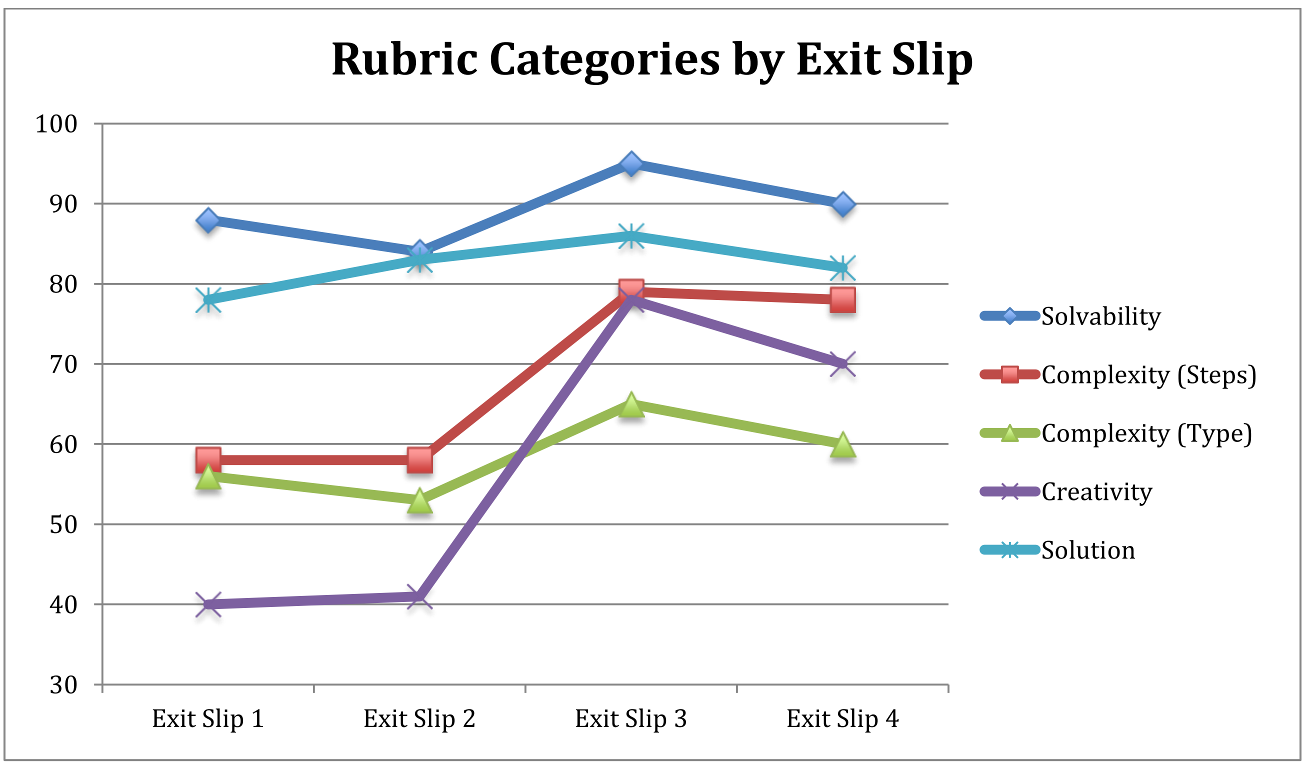


*Figure 7.* Line graph showing the average score on each rubric category on each exit slip

Growth for each category was determined by subtracting the average score for Exit Slip 1 for that category from the average score for Exit Slip 4 in that category Figure 8 illustrates the growth (in percentage points) for each rubric category over the course of the study.

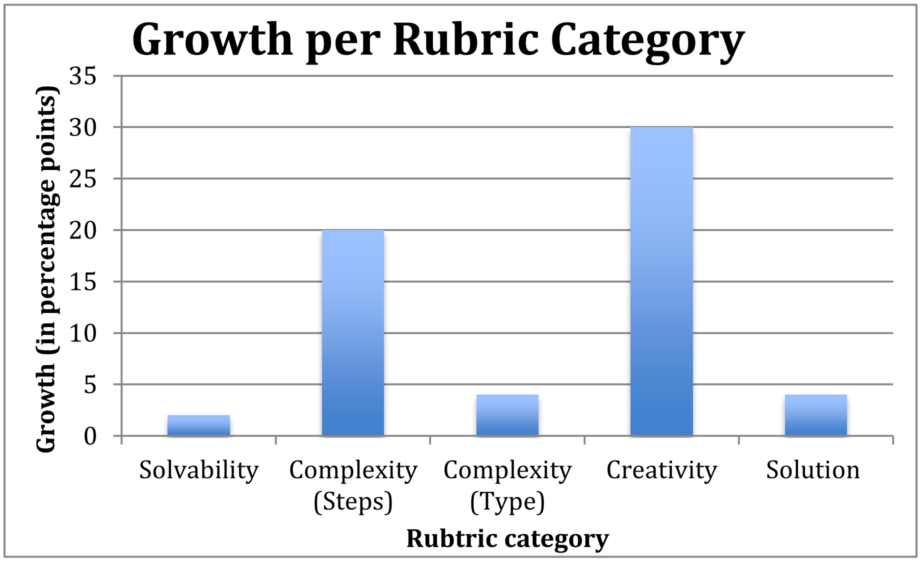


*Figure 8.* Bar graph showing percentage point growth in each rubric category

The categories with the least growth were Solution and Solvability. Scores in the Complexity (Type) category also did not see much growth throughout the duration of the study; they stayed moderately low (in the low sixties). The Complexity (Steps) category and the Creativity category both saw significant growth over the course of the four exit slips. Complexity (Steps) had a gain of 20 percentage points from Exit Slip1 to Exit Slip 4 (58% to 78%), and Creativity had a gain of 30 percentage points (40% to 70%).

Although the Complexity (Steps) and the Creativity categories saw significant growth across the four exit slips, the pre- to post-test growth was not as significant. Figure 9 shows the average scores for Complexity (Steps) and Creativity across all six weeks of the study.

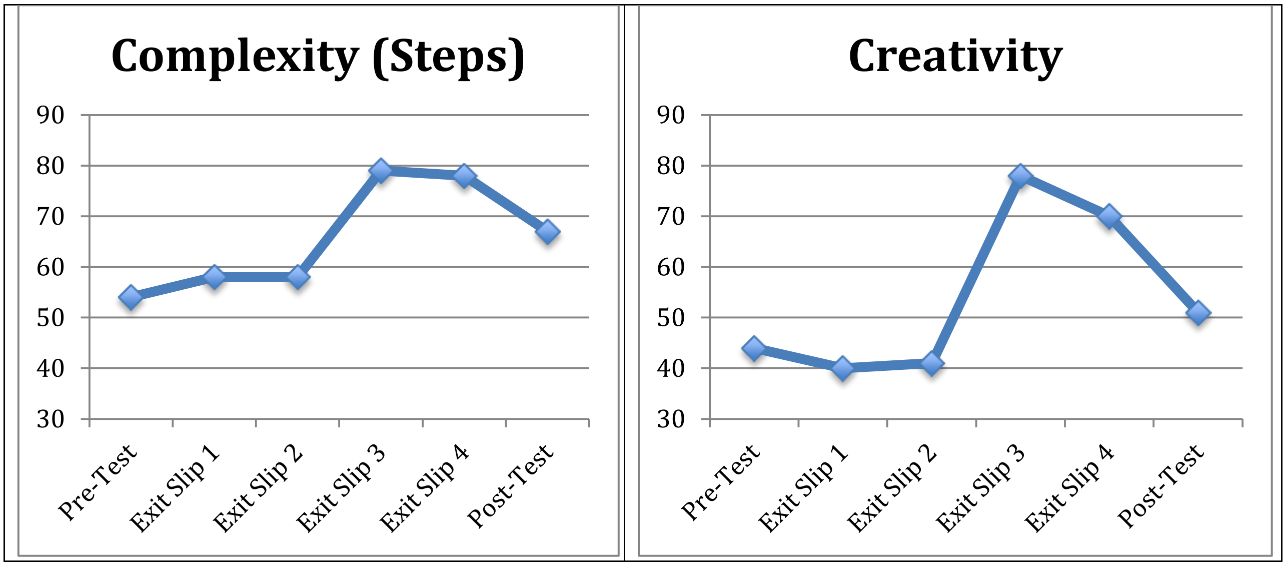


*Figure 9*. Line graphs showing the average scores for Complexity (Steps) and Creativity

In both of these categories, the most significant growth occurred on Exit Slip 3. And in both categories the post-test score was lower than the scores for Exit Slips 3 and 4. That being said, there was some enduring growth in these two categories from the pre- and post-tests. In the Creativity category the average score rose seven percentage points (from a 44% to a 51%). In the Complexity (Steps) category, the average score rose 13 percentage points (from 54% to 67%).

**Student creativity growth**. Although the overall scores for the Creativity category increased over the course of the study, students' growth in this category varied quite a bit. As the study progressed most students began writing more creative story problems. They began moving away from simple, single step story problems, toward more advanced multi-step problems. For example, on Exit Slip 1 Student #14 wrote a single step addition problem:

Joe went to Burger Land. Joe got a triple mean Burger, fries and a small drink. How much did it cost?

Solving the problem requires simply adding the price of the triple meat burger, the fries and the small drink. On Exit Slip 4 Student #14 posed a problem that requires three separate mathematics steps in order to solve.

> Joe's mom told him to go buy 5 box's of pasta and 6 boxs of ice cream. how much money does he need?

In order to solve this problem, you would need to: 1. Multiply the price of the pasta times five, 2. Multiply the price of ice cream times six and 3. Add those two products together. Although these two problems may not appear that different on the surface, they represent a major increase in complexity.

Student #17 also made significant growth in the area of Complexity (Steps). For the second prompt on the pre-test he wrote:

> Sonny ordered 3 large sodas how much do they cost?

His story problem for Exit Slip 4 was much more complex and required a lot more math.

> bob has $20.00 can he by 4 pastas 5 peanut butter and 2 icecreams?

On Exit Slip 1, Student #6 wrote a simple, straightforward story problem that required only addition of values directly from the provided menu:

> Three friends went to Burger Land. [Aaron] gets a cheese burger, [Katie] gets a Hotdog and [Molly] gets the Octo mean burger with chili, how much money did they spend?

On Exit Slip 4 Student #6 wrote much more creative problem. She introduced new information that is mathematically relevant for solving the problem – how much money the characters in the story had.

> 2 friends went to Wolly World. They counted up thier money and had 30$ total. They bought Brownie mix, Ice cream, pasta, penut Butter, yogurt and orange joice. Do they have enough money to by the items?

While the above examples illustrates positive growth over the course of the study, this type of growth was not seen for all of the students. Some students continued writing simple, non-creative problems throughout the entire study. For example, on Exit Slip 1, Student #4 wrote:

> One day I got a hamburger and a hot dog. How much did the food cost?

On Exit Slip 4 he wrote a problem that was very similar in terms of non-creativity. He wrote:

> I wanted Breyers [ice cream] Skippy peanut butter and Pillsbury flour. how much do they cost in all

Other students understood the importance of making creative problems, but got so caught up in the context of the story that the new information they introduced was not mathematically relevant. For example on Exit Slip 1, Student #10 wrote:

> One day a family of five was walking on the board walk in kings Dominion and they got really hungry so they went to Burgerland for lunch. It took them a while to deside what they were going to eat but then they found out. 2 people wanted cheese burgers and 3 people wanted Grilled cheese. How much money did they spend altogether?

Even after four workshop sessions, this student was still writing problems that were entertaining, but not mathematically creative.

> [Katie] and [Aidan] were making Brownies for there friend. While they were making the Brownies, Katie was about to put them in the oven when she dropped it on axadint! So we had to go to the store and get two more boxes of Brownie mix for are friend. How much money is in all?

For a few students, their scores in creativity declined over the course of the study. Students #7 and #16 wrote extremely creative problems on the pre-test. They thought way outside the box and wrote fascinating story problems. On the pre-test, Student #7 wrote:

> One time there was a bear in the woods. Mean while at the Foot ball game people were ordiring food… Jack ordered French fries and a cheesesteak. Bob ordered a hotdog, funnel cake fries, and popcorn chicken… The bear smelled the food so he ran east. Right after halftime they played the game again and the bear ran in the Football field and ran over footbal players and ate all the food. Jack and bob ran but then paid for all the foods on the menu. How may dollars did they have to pay? It turned out they had to pay 98$ and they ran without paying. Jack and bob were sent to catch the bear but never came back. Fox news said they were atacked bye a Bigfoot… the End

Student #7 did not even respond to prompt #4. His response to Prompt 3 took up the entire page so he just crossed out the final prompt.

Student #16 drew wrote a very creative problem with clever illustrations on his pre-test:

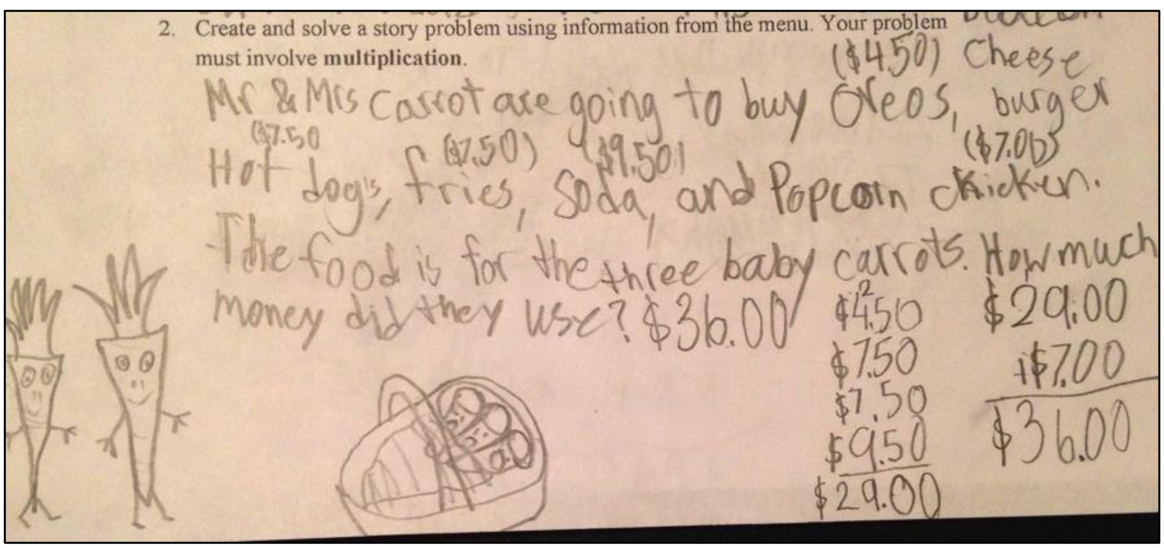


*Figure 10*. Photograph of Student #16’s response to Prompt 2 on the pre-test

As the project went on, both of these students lost some of their creative edge. They both wrote very straightforward problems on the post-test. On the post test Student #7 wrote

Bob bot a Hamburger and fries he had 10,00$ Does he have enough money?

Similarly, student #16 wrote:

[Eric] went to Sonny's. He got a Hot dog. Large french fries and a cheese steak. How much money did he spend?

**Prompt structure.** In order to fully assess how the complexity and creative of the problems students posed changed over the course of the study, it was also necessary to consider the various prompt structures that were present on the pre- and post-test and how those may have impacted performance. Figure 11 shows the average score on each of the prompts for both the pre-test and post-test.

On every prompt, the scores for the post-test are about 14 percentage points higher than those for the pre-test. Overall, the students made almost the exact same amount of growth on each style of prompt. On both the pre-test and the post-test students scored the highest on Prompt 2 and the lowest on Prompt 4.

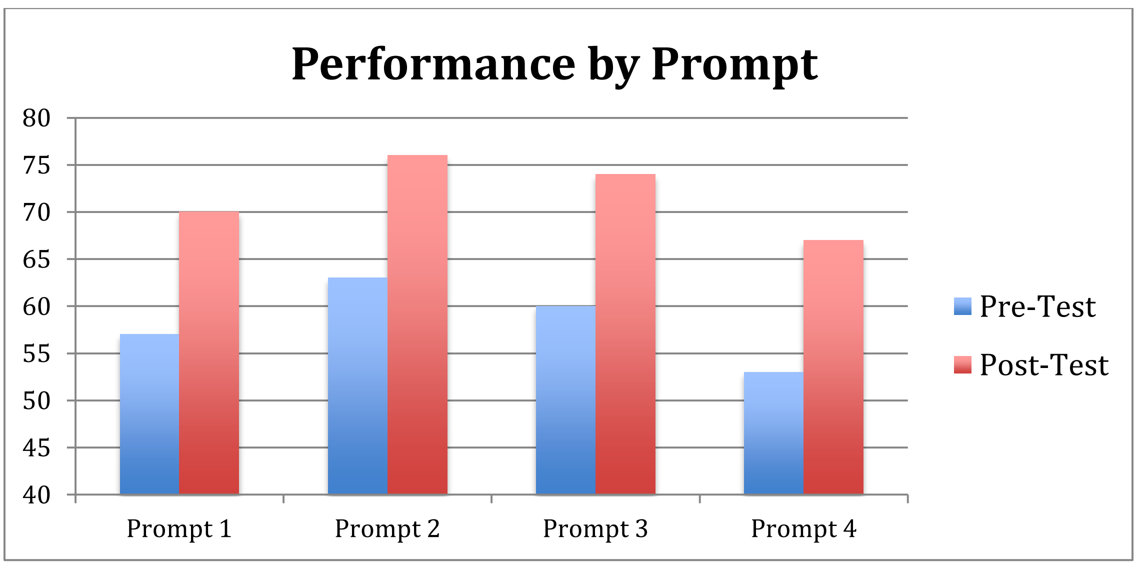


*Figure 11*. Bar graph showing average performance by prompt on the pre- and post test

It was clear from their responses, that students interpreted Prompt 4 in a variety of ways. Several students simply rewrote the same problem from Prompt 3. Some students partially understood the prompt; they did add details, but the additions were not mathematically relevant to the problem. Some students added descriptive details: Student #11 changed "One day me and my brother…" to "One *bright and sunny Wednesday* me and my brother…" Student #14 added the time of day: "Joey wen to Sonny's *at noon*. He got 15 hamburgers *for his family*." Other students wrote in the prices for each item instead of just the name of the item. On the post-test Student #8 wrote: Denis bug [bought] 2 hotdogs *that cost 3 bolers [dollars] each* and 2 funnel cakes *that cost 5 dollers each*. how much duse it cost in all?"

A few students were able to make mathematically relevant changes to their problem from Prompt 3. For the most part, these students simply added on additional item to the order they had come up with for Prompt 3. On the post-test Student #4 kept his problem from Prompt 3 the same but just added "and also combo 1." One student changed the payment method. He changed "how many would that cast?" to "how much pennys would it cast?" By changing the payment method to pennies, the mathematics needed to solve the problem also changed.

**Research Question 4: Thought Processes**

Students' responses in the exit interviews provided valuable insight into how the problem-posing process played out differently for different types of students. As Table 7

Table 7

*Overview of the students selected for exit interviews*

| **Student** | **Label(s)** | **Math Level** | **Notes** |
|---|---|---|---|
| Student #5 | ELL | Average | Quick learner, tends to rush |
| Student #7 | Gifted | High | Very creative, thinks outside the box |
| Student #9 | ADHD | Average | Visual, loves to build |
| Student #11 | | Average | Enjoys drawing, very imaginative |
| Student #15 | Gifted | High | Very sweet and well-behaved |
| Student #16 | | Average | Loves art, loves animals |

indicates, the students selected for the exit interviews represented a wide spectrum of learners.

When asked to 'tell me a little about' the problems they had created, the students provided a wide variety of responses. Students #9 and #5 both focused on the finished product and the mathematics they used in their problem. Student #9 explained, "So its two different ways….popcorn chicken and hamburger and you have to add those up to get the number." Student #5 said, "Um, this one. Here chart. So I say I buy 2 cheesesteak, 4 hot dog, 1 burger. How much money I have….how much money."

The other four students chose instead to discuss the process. Students #11 and #16 both emphasized how they were able to tailor the tasks to meet their own interests. Student #16 who loves to draw, explained, "In some of the problems, if I had space, I would draw the characters" The two gifted students, Student #7 and Student #15 both described how they attempted to exceed expectations. Student #7 said, "Well I guess I just tried to change up instead of just doing a regular problem. Like instead of doing like a boring one like cheese steak and plus funnel cake and plus soft drinks, I tried to do a fun one." He was interested in making an entertaining interesting problem. Student #15's motivation for exceeding expectations was a little different. She said, "Um, I wanted to use as many numbers as I could because I want to try to challenge myself kind of. And um, when I…like if has the small, medium, large, I like to use the large because it's a higher number." Her motivation was a bit more intrinsic than Student #7. This open-ended interview question allowed students to focus on whatever aspect of the process or product they wanted to.

Students' explanations of strategies and inspirations they used for problem posing, illustrate the flexibility and fluidity of problem-posing tasks. Students at a variety of academic levels were able to access these tasks. Students #9 and #5 had trouble explaining their problem-posing process. This type of reflective question was challenging for them. When asked how he decided what to make his story problem about, Student #9 shrugged and said, “I thought about it I guess. What do you mean?” Both of the gifted students discussed picturing their story in their head before they wrote. “To figure out I kind of picture in my mind what I want to do with all of this. And then I write it all down” (Student #15). Similarly, Student #7 added, “Like I make my story in my head before I write it.” When asked about the drawings he had made prior to writing his story problem Student #16 explained that he drew “To give me an idea about what to write.”

In terms of coming up with the story itself, students’ inspiration ranged from quite abstract to concrete immediate surroundings. Students #11 and #7, who are both very creative, emphasized the importance of considering the context where their story problem would take place. When asked about the context of her problem, Student #11 explained, “Because it – Burgeraland sounds like something familiar from one of my brother’s baseball games.”
And Student #7 said, “Well I just thought of a place that would have like a concession stand or something where you could get stuff from the menu.”

Students also mentioned the influence of their own experiences and background knowledge. Student #7 added, “I just figure out stuff I’ve seen on movies before and then stuff I’ve read in books before and I just kind of think of those and I just make up my own little story." Student #11 based one of her story problems on a story one of her friends had told her. “I was just thinking about how my friend went to the store and her mom gave her a specific list that she had to get and so like she bought like all the stuff that she wanted to get but she forgot about the list.” Student #16 mentioned an “eagle book” he had read and said, “I liked that book.”
A third type of inspiration was immediate surroundings – being inspired by what they actually saw when they were writing their problems – Student #16 explained that he wrote his problem for Exit Slip 3 about “Roby Robster” because, “I saw robins outside.” Similarly, #5 explained

why he chose to make one of his problems about a certain student in the class: “I decide that because I was looking at [him].” Only Student #9 was unable to explain how he came up with his story problems. He said, “I just wrote it I guess.”

The final two questions asked students if problem-posing tasks were harder or easier than what they usually do in math, and whether they were more fun or less fun. Three students (#7, #11 and #16) said problem-posing tasks were easier than regular math. Student #11 clarified, “Because um … because there’s not like … there’s a little bit of writing and there’s a little bit of math. So I don’t have to do everything about math.” For her, what made the tasks easier was that they were not solely about mathematics. She was able to do some writing during mathematics time and for her that made the tasks easier. Student #7 simply said, “I think they’re easier because I get to make up my own story.” For him the opportunity for creativity made the tasks easier – he was not as constrained. Student #16 explained that problem-posing tasks were easier “Cause I like solving um, cause I like solving multiplication and subtraction, addition and division problems.”

Student #9 and Student #15 both said the tasks were harder. Student #9 explained that these tasks are “[h]arder. Cause you have to write it out.” He added that they are harder “because you have to think about it and write it down and solve it. Instead of just solving it.” Student #15 also felt that these tasks are more challenging. She explained, “Because we have to…. Um, we don’t have the problem made for us here. We have to make it up and then solve it. Usually in class we have the problems made up for us and we just have to solve them.” She realized that there were more steps involved in these tasks: You have to write the problem and then solve it instead of just solving it. And student #5 said he didn’t know.

Five out of the six students said that problem-posing tasks are more fun than what they usually do in math. The students almost all emphasized the opportunity for creativity. "More fun…. Because I get to open up my mind … which I like to do" (Student #7), "Cause you get to be creative and write your own problem" (Student #9), and "They're more fun because I like being creative. I like making up my own stuff" (Student #15). Student #5 was hesitant to say that the tasks were more fun than regular math. He said, "Ah, I don't want to be like…I say my opinion. I think this one [problem posing], is better. Because, here is story you can solve it. You can make your *own* story. So you think in your brain and you make a story." Student #16 had slightly different rational. He emphasized his love of mathematics and drawing "More fun because I get to solve equations and draw pictures." When asked if the tasks were more or less fun than regular mathematics she said, "Um, kind of the same. Because I get to make up fun stories. But in math sometimes we get to build a lot of shapes."

The patterns and variety in the students' responses to the interview questions provided valuable insight into their thinking and problem posing process.

## Discussion

**Research Question 1:** *How do weekly problem-posing workshop sessions impact the performance of high and low ability problem posers?*

In looking at the performance of the low and high problem-posing groups over the course of the study, it is first interesting to note the academic diversity that is represented in each of these ability groups. Although some of the students in each group were students I would have expected to either struggle initially or excel right from the start, others were surprises. The fact that there were two gifted students in the low group, *only* two gifted students in the high group, and an ELL student in the high group, suggests that problem posing requires a set of skills distinct from those required for the general mathematics curriculum. The students who scored well on the pre-test were not necessarily the highest achievers in the class, and the students who initially struggled with problem posing were, in some cases, students who excel in other areas of mathematics. Problem posing appears to be a unique discipline, and performance does not align neatly with other academic labels or achievement categories. That being said, further research would be necessary to determine the specific nature of the relationship between problem-posing ability and other achievement variables.

Based on the findings discussed above, instruction in problem posing appears to have had a greater impact on the low group of problem posers than the high group. The high problem posers had fairly high scores from the start and did not make very much growth over the course of the study. Students #17 and Student #8 had post-test scores that were very similar to their pre-test scores; they continued writing very similar problems throughout the study and were resistant to trying new methods in their problems. Student #11 made negative growth from the pre-test to the post-test. Interestingly I think this may have been due to her attempts to make her problems more complex. As she attempted to add more outside information she lost sight of the mathematics involved and got carried away with the context. Instruction in problem posing was only beneficial to less than half of the high group. In contrast to the high group, all of the students in the low group made sizeable gains from the pre-test to the post-test. This suggests that instruction in problem posing may be more beneficial for students who are less experienced in problem posing. Perhaps these lower-performing students were more open to instruction than their higher-scoring counterparts.

Although the low group did make greater gains, it is also important to note that growth levels were also quite varied. This variation in growth may be the result of problem-posing performance being influenced by the structure of the assignment. The assessment measures I used may not have always provided an accurate reflection of students' problem-posing ability. Student #18's poor performance on the pre- and post-test, and her moderate-to-high performance on the exit slips suggests that the structure of the assessment played a role. Given her testing anxiety and how easily distracted she is, it makes sense that she was able to complete the single prompt exit slips but struggled with the four prompt pre- and post-tests. In her case the designation of 'low poser' is not entirely accurate. She was very capable of creating interesting mathematical problems but she lacked the focus and stamina to complete the lengthier assignments. This raises an interesting point for future research about how we score and assess students' abilities to pose mathematical problems. For Student #18 the structure of the pre- and post-test ended up assessing her stamina and focus rather than her problem-posing abilities.

Similarly, Student #13's performance was also likely impacted by the structure of the assessments. Because of his behavior issues and aversion to writing, he wrote as little as possible on his pre-test and consequently scored very low. He is certainly intelligent enough to write a complex, high-scoring problem but he did not want to. On the post-test I allowed him to dictate his story problems to me and I wrote for him. Although the problems he dictated were still not great, they were much more sophisticated than the problems he had come up with earlier in the study. Allowing him to dictate his story problems was a beneficial accommodation and allowed me to acquire a more accurate picture of his problem-posing abilities. Student #13's problem-posing growth was likely due more to the accommodation I provided for him on the post-test than to the instruction provided during the workshop sessions. The challenges that Student #18 and Student #13 had with the project illustrate the importance of accommodating individual needs. Without appropriate accommodations, students' performance may not provide an accurate picture of their abilities.

Overall, instruction in problem posing was more beneficial to the students in the low group than those in the high group. This finding is tempered by the fact that in some cases performance and growth were also impacted by the structure of the assignment.

**Research Question 2:** *To what extent is there a relationship between students' problem-posing and problem solving-abilities?*

Across the board, students scored higher in problem solving than problem posing. This is not surprising since elementary school students have much more experience solving pre-written story problems than solving their own. What is interesting to note is the lack of overlap, between the students who excelled or struggled in either problem posing or problem solving. Of the high problem solvers only one was also in the high problem-posing group, and of the low problem solvers, only two were also in the low problem-posing group. Several of the students who scored low in problem solving were able to do fairly well with problem posing. One of the benefits of problem posing is that students can access the tasks at a variety of entry points. Even students who are not strong in mathematics have the potential to write interesting problems. For example, Student #12 who was in the low group for problem solving (and is one of the lowest students in mathematics in the class), scored a 70% on the problem-posing post-test. Her low achievement in mathematics did not get in the way of her ability to pose interesting mathematical problems. Interestingly, though, as she began experimenting with more complex problems later on in the study, she often had trouble correctly solving her own story problems. Thus, she was able to write story problems but her lower problem-solving skills impacted her ability to solve the story problems she wrote.

Similarly, the students who scored the highest in problem solving were not necessarily the students who are the strongest in math. Student #10 and Student #16 are both average achievers in math, but because of their attention to detail and their strong work ethic, they both scored 100% on all four of the problem-solving task cards. Based on my observations as the student teacher, the students in the class who are the strongest in mathematics are also the students who tend to rush through their work, make careless mistakes and forget to show their work. Hence, many of the gifted students, despite their mathematical ability, were not in the high group for problem solving or problem posing. Student #11 and Student #8 were both in the high group for problem posing but not for problem solving. These students are low-to-average mathematics achievers and struggled with some of the problem-solving task cards. This lack of overlap between problem-solving and problem-posing ability, suggests that problem solving and problem posing require distinct skill sets.

Since the class average for problem solving was so high, it is also possible that these results are not completely reliable. Perhaps the single-step and multi-step problems were too easy and did not provide a full picture of students' problem-solving abilities. Additionally, the way the scoring rubric was set up, students had to show their work in order to receive full credit. This was a disadvantage for some of the gifted students who like to do all their work in their head. Having the rubric set up this way meant that in some cases I was assessing students ability to follow the directions rather than their ability to correctly solve the problem.

Based on my findings there does not seem to be a relationship between problem-posing ability and problem-solving ability. Since these findings are not generalizable, this would be a worthwhile question to explore further in future studies. It would also be interesting to conduct a more long-term study to examine whether or not extensive practice with problem posing would positively impact students' problem-solving performance.

**Research Question 3:** *How does weekly problem-posing practice impact the complexity and creativity of the problems students pose?*

Over the course of the study, scores improved significantly in both the Complexity (Steps) category and the Creativity category.  This suggests that, overall, weekly instruction and practice in problem posing positively impacts the complexity and creativity of the problems students pose. It is important to note that the growth in for complexity was in the Complexity (Steps) category rather than the Complexity (Type) category. For fourth grade students, whose mathematics experiences thus far has been almost exclusively in the four basic operations with whole numbers, it is understandable that they were not ready to write problems involving for complex mathematics (probability, fractions etc.). These higher-level mathematics skills would likely be more appropriate for the problem posing of middle or high school students. For fourth grade students, writing multi-step problems was an attainable goal, and the scores in the Complexity (Steps) category were positively impacted by instruction in problem posing.

Although there was substantial growth in the complexity and creativity of the problems students posed, there are some interesting caveats that seem to come along with this finding. One factor that appears to have influenced students' performance in writing complex and creative problems is the structure of the prompt itself. The pre- and post-test contained four different prompts that represented four of the prompt types discussed in the literature. On both the pre-test and post-test, students scored better on certain prompts and worse on others. Prompt 2 – which asked students to create a story problem involving multiplication – had the highest average score on both the pre- and post-test. This was interesting to me because as I observed the students taking the pre-test, Prompt 2 seemed to be as struggle for many of them. Some students skipped it and came back to it after they had finished the prompts. And other students raised their hand to ask what the question was asking. Perhaps since the problem dictated only a mathematical constraint and not a contextual constraint, students were more likely to introduce new information in their story problem (thus scoring higher in Creativity). Additionally, Prompt 2 asked students to involve multiplication in their problem so students tended scored higher in the Complexity (Type) category since including multiplication earns a higher score than just using addition and subtraction. So, perhaps the way the rubric was set up provided unreliable data regarding performance on Prompt 2.

The scores on Prompt 4 were the lowest of all four prompts on both the pre-test and the post-test. The prompt asked students to add to or change what they had written for Prompt 3. For most students, this idea of improving upon something they had already written was very new to them and (as discussed above) they interpreted the question in a variety of ways. Many of the students were initially confused by the question. During both the pre-test and post-test several students raised their hands to ask what to do for the last question. One flaw in the study was that the workshop sessions did not really address the skills required for Prompt 4. Students likely would have benefited from some direct practice with revising and expanding their problems. Perhaps this could be a fifth workshop session if the study were to be replicated in the future.

Since scores increased about the same amount on each prompt from the pre-test to post-test it can be hypothesized that instruction is beneficial but does not completely counteract the varying levels of difficulty that different types of prompts present. After four weeks of instruction, Prompt 2 was still the highest scoring, and Prompt 4 was still the most challenging. An additional factor that must be considered is that all four exit slips used open-ended prompts similar to Prompt 3 on the pre- and post-test.

In addition to prompt structure, it also appears that, in order for growth complexity and creativity to be fully retained, more practice would be necessary. Although the scores for Complexity (Steps) and Creativity, did increase from the pre-test to the post-test, the scores on the post-test were considerably lower than those on Exit Slips 3 and 4 (see Results section above). This may suggest that although students scored well on Exit Slips 3 and 4 (which were completed directly after the problem-posing workshop session on writing complex problems) these new skills were not fully integrated into students' problem-posing repertoire. Thus writing complex and creative problems is indeed possible elementary aged students but would likely require more substantial practice in order to be fully retained. It would be interesting to conduct a longer problem-posing study to explore the level of practice that would be necessary in order for students to fully integrate the skills of writing complex and creative problems.

One of the most interesting findings related to the creativity and complexity of the problems students posed, is that instruction in problem posing actually stifled the creative of students who were naturally the most creative. As they received instruction in problem posing, these students began to conform to the constraints and their problems became more mediocre. This suggests that for the highest, most creative students, instruction in problem posing is actually a detriment to their problem-posing performance. That being said, for the average students instruction in problem posing helped them be more creative. Additional research would be necessary in order to make any definitive conclusions about the relationship between problem-posing instruction and creativity.

**RQ 4:** *How do weekly problem-posing workshop sessions impact the thought process associated with problem generation?*

As the literature suggests, problem-posing tasks are a wonderful way to increase student engagement (Whitin, 2006). During my study, I found that students really did enjoy the problem-posing process. Although on the pre-test some students were initially a little confused, most of them quickly overcame their apprehensions and became very engrossed in completing the assessment. This suggests that, even without direct instruction or practice in problem posing, students naturally find these tasks to be engaging.

Students' high level of engagement with the process was further evidenced by their desire to share the problems they created with the class. Over the course of the project students became more excited about writing their own story problems and wanted to read me what they had written when they were finished. When time permitted I would allow a few students share their story problems at the end of the workshop session. Students really enjoyed this and were disappointed when we did not have time to share. As students became more comfortable with the process and began taking bigger risks to create more complex problems, they also became more eager to share their story problems with their peers and teachers. This suggests that weekly practice in problem posing increases engagement. Additionally, the fact that five out of six interviewed students said these tasks are "more fun" than what they usually do in mathematics is also evidence on students enjoying the tasks.

An additional explanation for increased engagement is that problem-posing tasks allow students to interact with content in ways that were personally meaningful to them. Students were able to work bits of personally relevant information into their story problems. This opportunity for personalization makes problem-posing tasks especially enjoyable for elementary students.

Of course, there was still variation in how students interacted with problem-posing tasks. Since the tasks were so open-ended, self-motivation played a huge role in determining the quality of the problems students posed. Many students saw the problem-posing tasks as an opportunity to be creative and challenge themselves. However, a few students remained uninterested throughout the duration of the study. For example, Student #4, a gifted student, did not put much effort into the assignments. He rushed through the exit slips and continued writing very simple problems even after four weeks of workshop session instruction. This variation suggests that while for most students, weekly practice in problem posing increases the levels of critical thinking that students exert in the problem-posing process, there are still some outliers who remain uninterested, despite the weekly workshop sessions. Weekly practice is an effective means for most, but not all students.

Despite the variation in self-motivation, overall, problem posing was something students really did enjoy. Students recognized the increased rigor of the task and appreciated the challenge. The fact that some of the students were better able to reflect upon the problem-posing process than others also raises some interesting questions for future research. In order to gain more definitive insight into the specific impact of instruction in problem posing, it would be useful for future studies to examine students' attitudes toward, and explanations of, problem posing both prior to, and after instruction. Conducting diagnostic interviews right after the pre-test would provide data that could later be compared to the responses in the exit interviews.

## Implications

Conducting this action research project was a wonderful experience. Through the process I learned so much, not only about problem posing in the elementary school classroom, but also about instruction and assessment in general. As I reflect upon conducting this study, there are some important lessons that I will bring with me into my first year of teaching.

**Open-ended mathematics tasks.** Having seen first-hand the numerous benefits of using problem-posing tasks in the classroom, I am eager to implement open-ended mathematics tasks with my future students on a regular basis. These tasks are not only engaging, but also provide a wonderful platform for differentiation. The class I implemented this study with was extremely diverse academically. The problem-posing tasks had the potential to challenge and engage every student in the class (including the level one ELL students, and the above-grade-level, gifted students) Students were able to pose problems at a variety of levels based on their competencies. The fact that a few of the high achieving students did not exert much effort, suggests that it might be useful to impose a few additional constraints in order to push them to reach their full problem-posing potential. Open-ended tasks based on the same general prompt could easily be differentiated to push the high achieving students who are not as self-motivated.

**Varying impact of instruction.** Although for most students, instruction in problem posing had positive impact on performance, there was still considerable variation in growth over the course of the study. In part this variation can be attributed to the fact that students entered the study with varying levels of background knowledge and skill. It is important to remember that even with the best instruction, external factors still play a role. Since students bring a wide range of experiences and skills to the classroom, the impact of any instruction with vary considerably by student. As a teacher it will be my job to do everything in my power to help my students succeed, but I will also need to remember that performance is influenced by factors beyond my control as well.

**Structure and context matter**. Through conducting this study, I saw firsthand how the structure of an assignment can negatively impact student performance. Students like Student #13 who are very averse to writing need to be given alternative avenues for self-expression. Allowing Student #13 to dictate his responses during the post-test gave him the opportunity to fully express his ideas and demonstrate his ability. When creating assignments and assessments I will need to be cognizant of how the structure of the task may impact my students' performance. Because there is always the potential for the structure of an assignment to influence outcomes, it is important not to use a quantitative score as the sole means to evaluate as student's performance. Using a combination of quantitative and qualitative measures provides a more well-rounded picture of students ability and is a helpful check of the validity of the assignment.

Additionally, it is important to consider that if a lesson is rushed or is right at the end of the day, student engagement and performance can be significantly affected. The workshop sessions that had to be squeezed in right after recess or right before PE did not go as well as those that were incorporated into mathematics time in the morning. When giving important assessments, it will be crucial for me to ensure they are given at a suitable time of day. It is also important to take these factors into account when scoring assessments. If students were distracted and preoccupied during a test, that will show in their work. In looking at students' performance on assessments it is important to keep these structural and contextual factors in mind.

**Talk to your students.** Through this study I also discovered the importance of talking to students about their work. I would have never understood the reasoning behind some of the interesting and very unique problems that students posed, if I had not taken the time to sit down and ask them to explain their thinking. It is often easy for teachers to make assumptions about students' work and their abilities and thinking. There is truly no substitute for asking students to explain their own work. For example, on his pre-test Student #16 wrote a story problem about

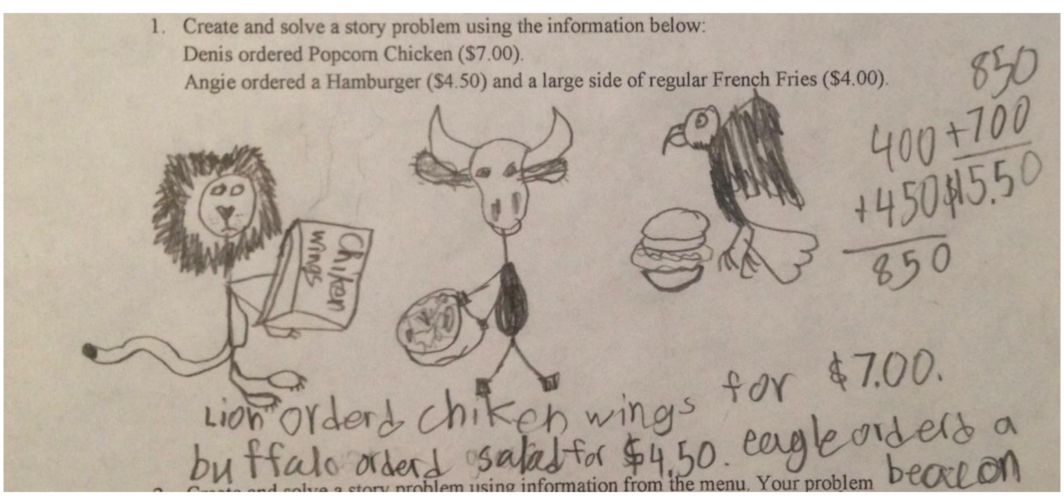


*Figure 12.* Photograph of Student #16's response to Prompt 1 on the pre-test

Lion, Eagle and Buffalo ordering food from the menu (Figure 12). I assumed he randomly chose what each animal had ordered, but when I asked him about the problem he explained that the lion and the eagle had both ordered meat dishes because they are carnivores, but the buffalo had ordered a salad because he his a herbivore. I would have never come up with that! This really illustrates the importance of talking to students about their work. Often there is so much more going on if I would just take the time to ask and listen.

**Beware of labeling.** An additional take-away from conducting this study is the danger of labeling students. One of the most compelling findings of this research was that the students who exceled or struggled with problem posing were not necessarily who I would have expected. It is so important for teachers not to assume students will or will not be good at something based on their performance in another area. Each subject, each new unit, is a new frontier, and as a teacher I want to give each student the opportunity to wow me every step of the way.

Conducting this study was a wonderful experience, and I sincerely hope to have the opportunity to conduct further research on this topic in the future.

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**Appendix A:** Pre- and Post-Test

# Pre-Assessment


1. Create and solve a story problem using the information below:
   Denis ordered Popcorn Chicken ($7.00).
   Angie ordered a Hamburger ($4.50) and a large side of regular French Fries ($4.00).

2. Create and solve a story problem using information from the menu. Your problem must involve **multiplication**.


3. Create a story problem using any of the information in the menu. Solve your problem.

4. Rewrite the problem you wrote for question 3 to make it more detailed or complex. Solve your new story problem.

**Appendix B:** Exit Slips

# Exit Slip 1

Create an interesting word problem based on this menu

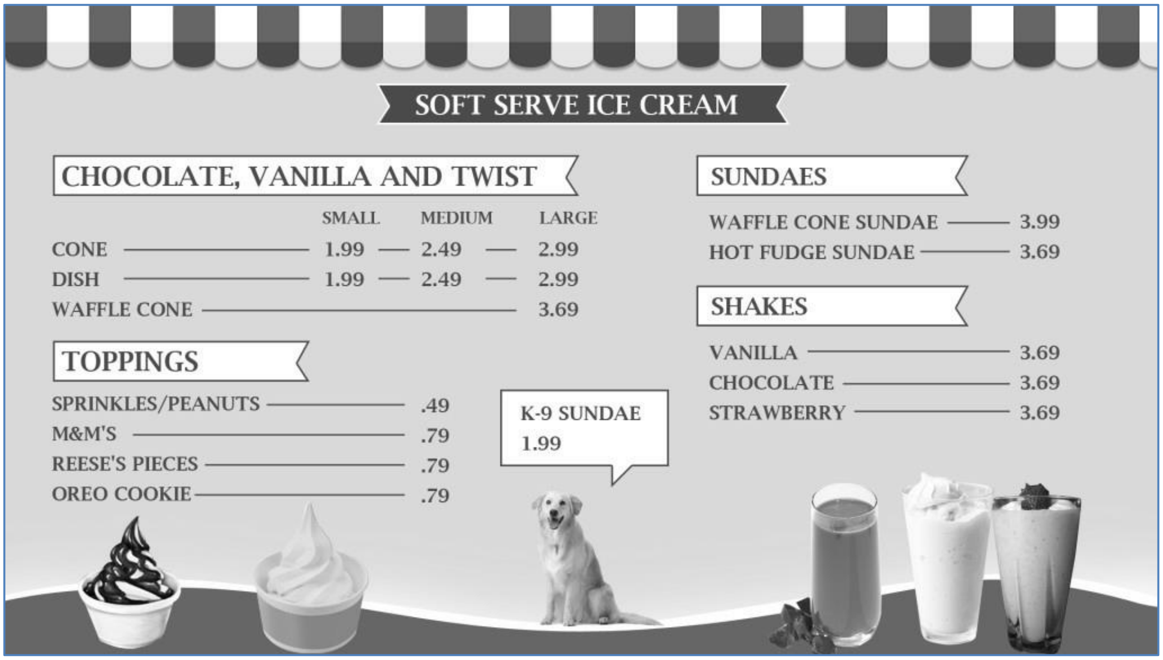

1. What information from the artifact did you use?

2. Write your problem:

3. Solve your problem:

# Exit Slip 2

Create an interesting word problem based on this menu




1. Write your problem:

______________________________________________________________________________
______________________________________________________________________________
______________________________________________________________________________
______________________________________________________________________________
______________________________________________________________________________
______________________________________________________________________________
__________________________________________________

2. Solve your problem:

# Exit Slip 3

Create an interesting word problem based on this price list


1. Write your problem:

________________________________________

________________________________________

________________________________________

________________________________________

________________________________________

________________________________________

________________________________________

________________________________________

2. Solve your problem:

# Exit Slip 4

Create an interesting word problem based on this menu


1. Write your problem:

______________________________________________________________________________
______________________________________________________________________________
______________________________________________________________________________
______________________________________________________________________________
______________________________________________________________________________
______________________________________________________________________________
______________________________________________________________________________
______________________________________________________________________________

2. Solve your problem:

**Appendix C:** Problem Solving Task Cards

Solve each problem. SHOW YOUR WORK!

Week 1:

1. Kira is packing 18 yo-yos in boxes. She packs them in 6 boxes with the same number in each box. How many yo-yos are in each box?

2. Ted, Bryce and Angelina are trying to raise $200 for a school fundraiser. Ted raised $32. Bryce raised $95. How much money does Angelina need to raise in order for them to reach their goal?

Week 2:

1. Jerry ran for 19 minutes 7 days in a row. How many minutes did Jerry run in all?

2. The total lunch bill for six people is $52. They add an $8 tip and then split the bill evenly. How much is each person's equal share of the total bill?

Week 3:

1. Four friends shared the cost of a boat ride. The total cost for the ride was $72. How much did each friend pay?

2. Jessie has 15 mystery novels, 8 fairy tale books and 7 comic books. She wants to display the books equally on two shelves. How many books should she put on each shelf?

Week 4:

1. Brett earned 230 points playing a video game. Jane earned 7 times as many points as Brett. How many points did Jane earn?

2. Celia has four weeks to save $58 for her vacation. In her first week, she saved $10, the second week $21, and in the third week $17. How much more does she need to save?

**Appendix D:** Parental Consent Letter

Dear Parent or Guardian,

My name is Gemma Cohen, and I am the student teacher in Ms. Wineski's fourth grade class this semester. I am currently a graduate student at the University of Mary Washington working towards my master's degree in elementary education. A requirement of the program is to conduct an action research study in an area related to our studies. *I am inviting your child to participate in the research study I am conducting. Involvement in the study is voluntary, so you may choose whether or not to have your child participate. Below is an explanation of the study.*

*I am interested in learning about how instruction in problem posing influences students' problem solving skills and their ability to create their own story problems.* The term problem posing refers to students creating their own mathematics word problems based on given information. Problem posing tasks help students apply and develop their critical and creative thinking skills

For four weeks, your child's class will be practicing different problem posing skills in order to improve their problem posing proficiency. *I am requesting permission to use your child's classwork as data for my study. I am also requesting to interview and tape record your child answering questions about the project. Interviews will last 5 to 10 minutes and will be conducted before or after school so that students do not miss any instructional time.* ***This project will be part of your child's work for class. It will in no way require extra work for him or her.***

Your child's work will be kept confidential. His or her name will not appear in any papers in the project. All names will be changed to protect his or her privacy. Following the project, all samples I collect will be destroyed. Participation in this project will not affect your child's grade in any way. His or her participation in the study is voluntary, and your child is free to stop participating in the study at any time. Your child would still complete the classwork components of the project, but data for the research study would not be collected from him or her.

*The benefit of this research is that you will be helping me understand the impact of problem posing practice on elementary students problem posing and problem solving abilities. The only potential risk is that your child may initially feel uncomfortable being interviewed. This risk will be minimized by conducting the interviews unobtrusively so that students won't feel singled out.*

If you have any further questions or concerns, please do not hesitate to contact me (gcohen@mail.umw.edu) or my university supervisor, Dr. Marie Sheckels (msheckel@umw.edu). Please return this form by January 12, 2014. I look forward to working with you and your student!

Thank you,

Gemma Cohen

I have read the above letter and give my child, ______________________________, permission to participate in this project.

____________________________________
(Parent/Guardian Signature)

I give my child permission to be tape-recorded during interviews.

______________________________
(Parent/Guardian Signature)

I, ____________________________ agree to keep all information and data collected during this research project confidential.

______________________________
(Researcher Signature)

**Appendix E:** Student Assent Letter

Dear Student,

I am very excited to be your student teacher this spring! For part of our math time, we will be working on getting better at writing our own story problems. We will do lots of practice and learn how to write different types of problems

During math I will be collecting information for a research project that I am doing. I want to see how practicing writing word problems helps you write better problems. During my study, I will interview you about the story problems you create and how you like the project, and I may tape record you to remember what you say. You will not be graded for your help in my study, and this study will not require you to have extra work. The only things you will do are talk with me about the work you have already done.

*Your parents were given a letter about taking part in this study. If your parents did not allow you to participate in this study, you will not be asked to sign this form. However, if your parents did allow you to participate, I encourage you to participate in this study.*

*You do not have to be in this study. No one will be mad at you if you decide not to do this study. Nothing bad will happen if you take part in the study and nothing bad will happen if you do not. However, if you decide not to participate you still will work in groups and do all of the work that we will do; I will just not use your work in my research. Even if you start, you can stop later if you want. You may ask questions about the study.*

*If you decide to be in the study, I will keep your information confidential. This means that I will not use your names or the name of the school in anything I write and I will not reveal any personal, identifying information about you.*

*Signing this form means that you have read it or have had it read to you, and that you are willing to be in this study.* If at any point you have any questions, please ask me!

Thanks,

Gemma Cohen

I have been read the above letter, all my questions have been answered, and I agree to participate in the project.

______________________________ ______________________________
(Student Signature) (Date)

I agree to be tape-recorded during interviews.

______________________________ ______________________________
(Student Signature) (Date)

I, ______________________________ will keep your names confidential.

______________________________ ______________________________
(Student Teacher/Researcher Signature) (Date)

**Appendix F:** Interview Questions

1. Tell me about the problems you created?
2. What did you do first?
3. How did you decide what to make your problem about?
4. How did you know you were done?
5. Are activities like this harder or easier than what you usually do in math? Why?
6. Are activities like this more fun or less fun than what you usually do in math? Why?

**Appendix G:** Scoring Rubric

| | 4 | 3 | 2 | 1 |
|---|---|---|---|---|
| **Solvability of the problem posed** | The problem posed is mathematical and contains sufficient information for solving | The problem posed is mathematical but is missing one piece of information for solving | The problem posed is mathematical but lacks more than one piece of information for solving | The problem posed is not mathematical |
| **Adherence to the prompt** | | The problem posed adheres to all the constraints outlined in the prompt. | The problem posed adheres to some, but not all, of the constrains in the prompt. | The problem posed is completely unrelated to the prompt. |
| **Complexity: # of Steps** | The problem posed requires three or more math steps. | The problem posed requires two math steps | The problem posed requires only a single math step | The problem posed does not require any math steps. |
| **Complexity: Problem Type** | The problem posed includes at least 1 math concept beyond the four basic whole number operations (e.g. fractions, ratios probability, etc) | The problem posed includes multiplication and/or division (exclusively *or* in addition to addition and/or subtraction) | The problem posed includes only addition and/or subtraction | The problem posed does not include any math |
| **Creativity** | The problem introduces new information mathematically necessary for solving the problem. | The problem uses the existing information in a new way that is mathematically relevant. | The problem introduces new information, but it is not mathematically relevant. | The problem only uses information directly provided in the prompt |
| **Solution to the posed problem** | The problem is solved correctly. All work is shown and the student's thinking can be followed. | The problem is solved correctly but the answer is incorrect due to one or more arithmetic errors *Or* the answer is correct but not all work is shown. | The problem is solved incorrectly (wrong operation etc.) but work is shown. | The answer is incorrect and work is not shown. |

**Appendix H:** Data Tables

**Pre-test** performance by rubric category

| Student | Solvability | Adherence | Complexity (Steps) | Complexity (Type) | Creativity | Solution | Score |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 0.75 | 0.5 | 0.5 | 0.25 | 1 | 17% |
| 2 | 3.75 | 2.75 | 2.5 | 2.75 | 1.75 | 3 | 72% |
| 3 | 1.75 | 1.75 | 1.25 | 1.5 | 1.75 | 1.5 | 41% |
| 4 | 3.5 | 3 | 2.25 | 2.25 | 1.75 | 2 | 64% |
| 5 | 3.25 | 2.75 | 2.25 | 2.5 | 1 | 2.75 | 61% |
| 6 | 4 | 2.75 | 2.25 | 2 | 1 | 3 | 65% |
| 7 | 1.75 | 1.5 | 1.75 | 1.5 | 4 | 1.5 | 48% |
| 8 | 3.5 | 2.75 | 2.75 | 2.25 | 1.5 | 3.75 | 72% |
| 9 | 4 | 2.75 | 2.25 | 2.75 | 1 | 3.75 | 68% |
| 10 | 2.75 | 2.75 | 2.5 | 2.25 | 3 | 2.5 | 68% |
| 11 | 4 | 2.5 | 2.5 | 2.25 | 2.25 | 3.5 | 74% |
| 12 | 3.5 | 2.75 | 2.75 | 2 | 3.25 | 1.5 | 68% |
| 13 | 1 | 1 | 1.25 | 1.25 | 0.5 | 1.25 | 25% |
| 14 | 3.75 | 2.75 | 2 | 2.25 | 2 | 2.5 | 66% |
| 15 | 3.5 | 2.75 | 3.25 | 2.75 | 2.5 | 3.25 | 78% |
| 16 | 3 | 2.5 | 2 | 2 | 2 | 2 | 59% |
| 17 | 4 | 2.75 | 2.5 | 2.75 | 1.75 | 3.25 | 74% |
| 18 | 1 | 0.5 | 0.5 | 0.5 | 0.25 | 0.75 | 15% |
| **Avg:** | **74%** | **77%** | **54%** | **52%** | **44%** | **59%** | **58%** |

**Post-test** performance by rubric category

| Student | Solvability | Adherence | Complexity (Steps) | Complexity (Type) | Creativity | Solution | Score |
|---|---|---|---|---|---|---|---|
| 1 | 3.5 | 3 | 2.75 | 2.25 | 3.25 | 3.25 | 78% |
| 2 | 3.75 | 2.75 | 3.75 | 3 | 3.75 | 3.75 | 90% |
| 3 | 2.5 | 2.5 | 2 | 2 | 1 | 3.25 | 58% |
| 4 | 3.75 | 3 | 2.75 | 2.25 | 1.75 | 4 | 76% |
| 5 | 3.5 | 3 | 2.75 | 2.5 | 1.75 | 3.5 | 74% |
| 6 | 3.5 | 2.75 | 2.25 | 2.25 | 1.5 | 3.75 | 70% |
| 7 | 3.75 | 2.5 | 2.75 | 2 | 3 | 3.5 | 74% |
| 8 | 4 | 2.5 | 3 | 2.5 | 1.5 | 3.25 | 71% |
| 9 | 3.75 | 3 | 3.5 | 3 | 2.75 | 3.75 | 86% |
| 10 | 4 | 2.75 | 2.5 | 2 | 2 | 3.75 | 74% |
| 11 | 3.5 | 2.5 | 2 | 2 | 1.75 | 2 | 60% |
| 12 | 3 | 2.75 | 3.25 | 2.5 | 2 | 2.5 | 70% |
| 13 | 2.5 | 2.5 | 2.75 | 2.25 | 1 | 3.25 | 62% |
| 14 | 3.75 | 3 | 2 | 2.75 | 1.75 | 4 | 75% |
| 15 | 4 | 3 | 3.75 | 2.75 | 3.5 | 4 | 91% |
| 16 | 4 | 2.75 | 2.5 | 2.25 | 1.25 | 4 | 73% |
| 17 | 3.25 | 3 | 2.5 | 2.75 | 2.75 | 3 | 75% |
| 18 | 2 | 1.25 | 1.75 | 1.5 | 0.75 | 1.5 | 38% |
| **Avg:** | **85%** | **90%** | **67%** | **59%** | **51%** | **83%** | **72%** |

Summary of Individual and Class Average Gains

| Student | Pre-Test | Exit Slip Average | Post-Test | Pre-to Post gain |
|---|---|---|---|---|
| 1 | 17 | 78 | 78 | 61 |
| 2 | 72 | 69 | 90 | 18 |
| 3 | 41 | 59 | 58 | 17 |
| 4 | 64 | 68 | 76 | 12 |
| 5 | 61 | 88 | 74 | 13 |
| 6 | 65 | 70 | 70 | 5 |
| 7 | 48 | 73 | 74 | 26 |
| 8 | 72 | 68 | 71 | -1 |
| 9 | 68 | 73 | 86 | 18 |
| 10 | 68 | 84 | 74 | 6 |
| 11 | 74 | 68 | 60 | -14 |
| 12 | 68 | 61 | 70 | 2 |
| 13 | 25 | 36 | 62 | 37 |
| 14 | 66 | 74 | 75 | 9 |
| 15 | 78 | 83 | 91 | 13 |
| 16 | 59 | 75 | 73 | 14 |
| 17 | 74 | 79 | 75 | 1 |
| 18 | 15 | 73 | 38 | 23 |
| Class Average: | 58% | 71% | 72% | 14% |